

A Decision Support Model for Real-Time Quality Control and Logistics Tracking under Spherical Fuzzy Aczel-Alsina Aggregation Information with Z-Numbers

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ABSTRACT

This research presents a new method for real-time logistics monitoring and quality control in supply chain management using spherical fuzzy Z-number Aczel-Alsina (AA) aggregation. A method based on quantum cognitive theory is employed to improve the accuracy and efficiency of logistics tracking. The proposed method provides a robust framework for decision-making when dealing with dynamic and unpredictable real-time logistics data. The results demonstrate improvements in supply chain transparency and logistics tracking accuracy, which are important for timely and informed logistics management decisions. This research employs a multi-attribute group decision-making (MAGDM) approach to address this complex problem. The main aim of this paper is to propose and explore spherical fuzzy Z-number sets (SFZNSs). Compared with existing fuzzy set structures, SFZNSs provide a more flexible framework for representing higher levels of uncertainty. We introduce an innovative distance measure for spherical fuzzy Z-numbers and derive unknown weight information using spherical fuzzy entropy calculations. Additionally, aggregation operators based on the AA t-norm and t-conorm are introduced to address MAGDM problems within the spherical fuzzy Z-number environment. The proposed method is particularly useful when the weights of the criteria and decision makers are unknown. By integrating AA operations within the spherical fuzzy Z-number framework, the proposed methodology provides a flexible and robust approach to information aggregation, enabling complex and uncertain information to be effectively represented and processed in decision-making problems.

1. Introduction

At any given time, an observer is needed to avert entropy, which has beset many organizations over the years. Because of this reason, companies must be able to track their products from production through distribution until they reach the final consumer. By monitoring logistics in real time, however, suppliers address defects by guaranteeing visibility into transport systems as well as managing resource allocation and flow of data across various channels. This monitoring includes col-

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lection and analysis of information such as where a shipment is located at a given point in space-time (geographic coordinates); dynamically changing percentages of stocks held at different warehouses (inventory levels); historical as well as current routes taken by transit agents (transit routes); planned movement of goods over predefined time periods (delivery schedules); Internet of Things (IoT) sensors installed on trucks and containers; geographic positioning systems (GPS) fixed onto vehicles carrying freight etc. This enables sending precise information, which aids better decision-making processes. Consequently, the company would spend less money on resources resulting into lower prices for its products, hence avoidance of interruptions that may be prompted by customer dissatisfaction. Owing to a globalized economy characterized by fast responding supply chains that are intertwined with consumer preference changes; this issue becomes particularly pertinent. According to literature, real-time logistics tracking provides real-time information about item flow, warehousing levels and transportation paths [1]. The ability to see things as they happen is indispensable in making good choices. For instance, procurement, sales and logistics teams can access this information through a collective decision-making process [2]. This shared visibility enables all team members to have the same information when making choices, which reduces ignorance and confusions. Supplier selection processes commonly utilize MAGDM models where various factors such as price, standards and reliability are considered [3]. Real-time logistics control makes it possible to get supplier performance data including adherence to delivery schedules and quality of products. This kind of data can then be included into MAGDM models for supplier selection and ranking in terms of their performance with respect to multiple criteria.

The models of MAGDM enable a thorough inspection of performance of suppliers and logistics, taking into account various factors. MAGDM models improve decision-making accuracy by integrating real-time data. Also, this approach ensures that many interests are considered through cooperative decision-making. On the other hand, there are some disadvantages of MAGDM models for the outlined context. Sometimes, it takes a lot of time to implement models because they need high-performance computers and techniques. Furthermore, a huge amount of data is generated by real-time logistics tracking, which may be difficult to process effectively and integrate [4].

Traditional MAGDM models may struggle to adapt rapidly to dynamic supply chain changes. Traditional decision-making approaches often fall short in capturing the intricate dynamics and multifaceted nature of real-time logistics tracking. The pervasive unpredictability and complexity [5] inherent in the decision-making environment pose significant challenges to decision-makers. Consequently, there is a pressing need to develop more flexible and adaptive decision-making. The motivation for this research stems from the necessity to enhance traditional methods by incorporating fuzzy set-based decision-making approaches. Utilizing intuitionistic fuzzy set (IFS) with the Aczel-Alsina (AA) [6] operator offers a more comprehensive and adaptable decision-making process that accommodates uncertainty and imprecision, leading to optimal solutions are represented in Table 1. This study aims to provide a more robust and effective framework for addressing Multi-Attribute Decision-Making (MADM) challenges in real-time logistics tracking, ultimately improving operational efficiency, reducing costs, and minimizing disruptions.

Table 1
Superiority of *SFZN* for real time logistic tracking over other fuzzy sets

Sets	Membership	Non-membership	Neutrality	Reliability
FS [7]	✓	×	×	×
IFS [8]	✓	✓	×	×
SFS [9]	✓	✓	✓	×
<i>SFZN</i>	✓	✓	✓	✓

To address this difficulty, Zadeh [10] introduced the innovative concept of fuzzy sets (FSs) in 1965.

The concept of fuzzy sets, which permits membership degrees to be assigned from [0,1], has developed into a foundational concept with significant ramifications across several scientific and technological fields. Zadeh [11] claims that groundbreaking research in this area has had a significant influence because researchers have found that fuzzy sets have a wide range of applications in several industries. Experts in the intricate realm of decision-making place a lot of emphasis on determining "membership and co-membership grades," a nuance that fuzzy sets (FS) struggle to capture. Thus because if we look at things this way it was Atanasov [12] who came up with the term intuitionistic fuzzy set (IFS). FS and IFS are interlinked devices for dealing with the complicated dilemmas of ambiguity. Often, these problems have their roots in the flawed parameters estimation techniques. In conditions where total number of membership degrees exceeds unity, reaching a conclusion within the confines of the IFS could be daunting. Additionally, this approach has its limitations since human beings tend to hesitate and control themselves. To bridge this gap, Hussain *et al.* [13] improved upon the IFS. In addition to that, Cuong and Kreinovich [14] made another important step forward by presenting an idea for image fuzzy sets (PiFS). The three factors that account for these PiFS are $\sigma(\varpi)$, $\mu(\varpi)$, and $\varphi(\varpi)$ which represent degrees of satisfaction, neutrality and discontentment at various levels respectively. Making $0 \leq \sigma(\varpi) + \mu(\varpi) + \psi(\varpi) \leq 1$ is a critical constraint placed on PiFS. Cuong's invention of PiFS became popular, however because of the inherent criteria on $\sigma(\varpi)$, $\mu(\varpi)$, and $\psi(\varpi)$, it did put certain limitations on decision-makers when assigning values. As a result, circumstances developed where the total cumulative sum of membership degrees was more than unity, resulting in illogical results.

A strong answer to this problem was provided in 1999 by the erudite Russian researcher Molodtsov [15], who introduced the idea of "soft sets (SS)". This novel concept, which constitutes a new and separate class of classifications in the field of considerations, has shown to be a flexible instrument in a number of fields. Its real-world applications are many and include, but are not limited to, decision-making, Riemann integration, game theory, measurement theory, and function smoothness assessment. When Jiang *et al.* [16] brought entropy into the domain of intuitionistic fuzzy soft sets (IFSS), this landscape was expanded.

Although it is simple to understand the linear difference between "membership grade," denoted by σ , and "non-membership grade," denoted by ψ , both the FSS and IFSS frameworks struggle to cope with flaws and ambiguities. Situations where decision-makers choose MG and NMG values of 0.5 and 0.7, respectively, pose a problem in the context of the IFSS framework since they violate the constraint $0.5 + 0.7 > 1$. Wang and Khalil [17] proposed a generalized Pythagorean fuzzy soft set (PFSS) and demonstrated its applicability in decision-making problems. The idea of "Pythagorean fuzzy sets (PFS)" [18] as an efficient way to overcome this limitation, leading to the creation of the novel concept of PFSS. The essential limitations were skillfully reformulated by these academics from $0 \leq \sigma + \psi \leq 1$ to $0 \leq \sigma^2 + \psi^2 \leq 1$. Because of their structural bases, FSS and IFSS are clearly unique cases of PFSS. Though helpful in addressing many important MADM problems, the PFSS framework is not perfect [19]. Atanassov's design is inadequate in light of the complexity of human behavior, which includes aspects of hesitation and abstention. It mainly considers the degrees of satisfaction and discontentment inherent within a collection of elements.

In order to increase the dimensional expanse for membership gradations, Ashraf [20] cleverly developed the concept of spherical fuzzy sets (SFSs), which are represented by the symbols $\sigma(\varpi)$, $\mu(\varpi)$, and $\psi(\varpi)$, which stand for membership, neutrality, and non-membership grades, respectively. Essential limitations upon these sets are provided by the formula $0 \leq \sigma^2(\varpi) + \mu^2(\varpi) + \psi^2(\varpi) \leq 1$, which also allows for countless more human preferences and abilities. SFSs, which have lately returned to the scientific agenda, account for the ever-increasing complexity of humans. Finding the center and surrounding radius of an SFS is necessary before building it. The former indicates its most significant region, while the latter indicates the portion of the map that includes every member. Every component is included within this radius that radiates out from the center; if it is not, its significance decreases with increasing distance. The distance between an individual and this center determines their membership status.

This method is innovative and suitable for complex systems where the interaction of different information sources may take place as it shows complexity, flexibility, and an enhanced uncertainty representation. Systematic fuzzy sets (of SFSs for short) preserve relationships and allow interactions among entities rather than isolating them thus providing connectives across the members of the class in question; hence they are more effective at uncovering ambiguity when meaning extends across dimensions. In 2011, Zadeh [21] proposed the revolutionary concept of z -number to further improve fuzzy sets. It changed from ordinary fuzzy figures into a condition of adaptation. By relating pairings of sequences with fuzzy number based on harmonics it is possible to establish a common basis related to how humans think about things which allows us to prove their existence. This alternative approach to traditional fuzzy set paradigms could assist with an understanding of reliability that is crucial when making tough decisions in today complex world. For example, since z -number are used continuously in many disciplines, they are sometimes regarded as a universal language. In cases of ambiguity and insufficiency, z -number may accurately reflect data dependability when it comes to measuring uncertainty. An outstanding example of this is Shannon's entropy, which may be used to assess uncertainties that adequately handle issues coming from constraints and questions about data gathering methodologies [22]. Frequent use of z -number facilitates precise estimate of uncertainty ranges with respect to complexity and enhances communication.

The unique notion of z -number was created to expand and improve the fundamental concepts of fuzzy number into this specialization of a particular class of ambiguous number, based on considerable study conducted by Banerjee [23]. These number have certain unusual properties under certain specified circumstances that may be used to solve a variety of issues under unpredictable circumstances. A whole sphere of uncertainty has been condensed into a tidy structure. This extensive study illustrates applications for z -number in several fields, including pattern recognition, application technology systems, and high-level field decision-making. In fact, this work provided insightful information on the use of z -number in both theoretical and practical contexts.

Similarly, Ashraf [24] introduced the notion of spherical fuzzy z -number sets ($SFZNS$) with the following properties: $0 \leq \sigma^2(\varpi) + \mu^2(\varpi) + \psi^2(\varpi) \leq 1$; $0 \leq \tau_{\sigma^2(\varpi)} + \tau_{\mu^2(\varpi)} + \tau_{\psi^2(\varpi)} \leq 1$. The degrees of satisfaction, detachment, and dissatisfaction are represented by the values $\sigma(\varpi)$, $\mu(\varpi)$, and $\psi(\varpi)$; the guiding values $\tau_{\sigma(\varpi)}$, $\tau_{\mu(\varpi)}$, and $\tau_{\psi(\varpi)}$ serve as indicators of the dependability of the aforementioned levels of contentment, abstention, and discontentment.

Aczél and Alsina [25] characterized several classes of quasilinear functions and established their applications to triangular norms and the synthesis of judgments. In addition, we have presented aggregation operators and the AA [26] operation in the context of spherical fuzzy z -number. Here, as mentioned in [27], the incorporation of spherical fuzzy z -number assumes paramount significance, as it capitalizes on the latest developments in fuzzy sets, systems, and decision-making. Owing to the inherent adaptability of AA t-norms and t-conorms, a number of researchers have focused on developing a range of aggregation operators based on these operations in various fuzzy contexts. Examples of these include some Maclaurin symmetric mean aggregation operators based on AA operations for IFS and their use in decision-making [28], and IF averaging AA [29] operators based on interval-valued IFS and their applications. Furthermore, approaches to multiple attribute group decision-making based on IF power aggregation operators [30] and they find use in centroid coordinate ranking of Pythagorean fuzzy number and its application in group decision-making [31]. The idea of power aggregation operators [32] inside a two-dimensional linguistic IF framework. Application of geometric aggregation operators to MADM via analysis of interval-valued IF AA [33]. Multiple-attribute decision-making using fuzzy AA [34] aggregation operations that are complex and IF.

Recent studies have increasingly explored Aczél–Alsina (AA) aggregation operators for handling complex and uncertain information in MCDM problems. For instance, Karamat and Sarfraz [35] proposed prioritized AA aggregation operators for complex Pythagorean fuzzy data and demonstrated their applicability to a software-company decision problem. These recent developments highlight the flexibility and effectiveness of AA operators in different fuzzy decision-making environments.

The rationale behind formulating the proposed *SFZN* aggregation operator rooted in the AA T-norm and T-conorm stems from the aspiration to enhance the effectiveness of decision-making procedures within fuzzy systems. The AA T-norm itself serves as a robust tool for calculating the intersection of fuzzy sets. The development of a novel aggregation operator based on the AA T-norm presents an opportunity to tailor decision-making processes, enhance performance, address research gaps, accommodate more intricate data types, contribute to theoretical advancements, conduct comparative analyses, and introduce innovation into the realm of fuzzy systems. The proposed aggregation operator facilitates comparative assessments alongside existing operators, enabling researchers to evaluate its merits and shortcomings in relation to other methodologies.

The research proposal outlines the following primary objectives:

- To explore the adaptation and performance of AA T-norms and T-conorms within the context of Spherical Fuzzy *Z*-number.
- To define and comprehensively examine the AA aggregation operations in the *SFZN* setting, assessing their key properties to extract valuable insights.
- To elucidate the foundational principles of AA T-norms and T-conorms within the *SFZN* framework, fostering a deep understanding of their underlying concepts.
- To conduct a comparative analysis, delineating the strengths and limitations of the proposed AA aggregation operators in the realm of *SFZN* in contrast to existing aggregation operators.

The article is organized in the following manner: Section 2 offers an introduction to basic preliminary processes like related operators, scoring methods and accuracy functions. In section 3 presents a contemplation on AA operators applied for *SFZN*. *SFZN* are given attention in section 4 with their proofs and important properties that accompany them. The methodology for MADM is outlined in section 5, *SFZN* while case study about real-time supply chain management logistics monitoring and quality control are delved into in 6 section. Section 7 presents the results and discussions. Moreover, a comparative analysis of *SFZN* aggregation operators on AA operators versus other *SFZN* operators is contained within this segment of this article. Finally, the last section 8 of this written composition consist of a conclusion related to all these discussions made in various sections. The abbreviations of full forms are given in Table 2.

2. Preliminaries

In order to construct the suggested tasks, this part presents a number of basic concepts and operations. These criteria provide the foundation for our system, guaranteeing sound and understandable theoretical foundations. By outlining these fundamental ideas, we provide readers the background knowledge they need to comprehend the advancements in our study that follow.

Definition 1. [10] The fuzzy set recognized within the whole set ν can be computed as follows:

$$\tilde{\nu} = \{(\chi, \varphi_{\tilde{\nu}}(\varpi) | \chi \in \nu)\} \quad (1)$$

where $\varphi_{\tilde{\nu}}(\varpi)$ degrees of contentment of ψ in $\tilde{\nu}$ and $\varphi_{\tilde{\nu}} : \nu \rightarrow [0, 1]$.

Definition 2. [21] An ordered duo of fuzzy number, represented by $Z = (\alpha, \tau\alpha)$, is called a *z*-number. The first component of the ordered pair is defined as the contentment which is represented by α while the second component is reliability, and it is represented by $\tau\alpha$.

Definition 3. [20] The SFS is identified under the entire set ν :

$$\tilde{\mu} = \left\{ \left(\chi, \left(\varphi(\varpi), \tau(\varpi), o(\varpi) \right) \right) | \chi \in \nu \right\} \quad (2)$$

such that $\varphi : \nu \rightarrow [0, 1]$, $\tau : \nu \rightarrow [0, 1]$, and $o : \nu \rightarrow [0, 1]$ are degrees of contentment, abstention, and discontentment, respectively, in a set ν , where:

$$0 \leq (\varphi(\varpi))^2 + (\tau(\varpi))^2 + (o(\varpi))^2 \leq 1 \quad (3)$$

Definition 4. [24] Suppose ν is the entire set. Then, *SFZN*s is identified as:

$$\mathfrak{S}_{\kappa} = \{ \psi, (\sigma, \tau_{\sigma})(\varpi), (\mu, \tau_{\mu})(\varpi), (\psi, \tau_{\psi})(\varpi) | \psi \in \nu \} \quad (4)$$

Table 2
Abbreviations and its full forms

Abbreviation	Full Form
AA	Aczel-Alsina
MAGDM	Multi-attribute group decision-making
SFZNS	Spherical fuzzy Z-number sets
IoT	Internet of Things
FS	Fuzzy set
IFS	Intuitionistic fuzzy set
PiFS	Picture fuzzy set
SS	Soft set
PFS	Pythagorean fuzzy set
SFS	Spherical fuzzy set
$SFZNWA_{AA}$	Spherical fuzzy Z-number Weighted Averaging Aczel-Alsina
$SFZNOW_{AA}$	Spherical fuzzy Z-number Ordered Weighted Averaging Aczel-Alsina
$SFZNHW_{AA}$	Spherical fuzzy Z-number Hybrid Weighted Averaging Aczel-Alsina
$SFZNWG_{AA}$	Spherical fuzzy Z-number Weighted Geometric Aczel-Alsina
$SFZNOWG_{AA}$	Spherical fuzzy Z-number Ordered Weighted Geometric Aczel-Alsina
$SFZNHWG_{AA}$	Spherical fuzzy Z-number Hybrid Weighted Geometric Aczel-Alsina
DM	Decision maker
GDIS	Group decision ideal solution
GRID	Group right ideal decision
GLID	Group left ideal decision
CI	Closeness indices

tableabb

such that $(\sigma, \tau_\sigma) : \nu \longrightarrow [0, 1]$, $(\mu, \tau_\mu) : \nu \longrightarrow [0, 1]$ and $(\psi, \tau_\psi) : \nu \longrightarrow [0, 1]$ are the order pair of degrees of contentment, abstention, and discontentment, respectively, in a set μ and second component is spherical measure of integrity for first component along all the conditions:

$$0 \leq \sigma^2(\varpi) + \mu^2(\varpi) + \psi^2(\varpi) \leq 1 \quad (5)$$

and

$$0 \leq \tau_\sigma^2(\varpi) + \tau_\mu^2(\varpi) + \tau_\psi^2(\varpi) \leq 1 \quad (6)$$

Definition 5. [24] Suppose $\mathfrak{S}_{\kappa_1} = \{(\sigma_1, \tau_{\sigma_1}), (\mu_1, \tau_{\mu_1}), (\psi_1, \tau_{\psi_1})\}$ and $\mathfrak{S}_{\kappa_2} = \{(\sigma_2, \tau_{\sigma_2}), (\mu_2, \tau_{\mu_2}), (\psi_2, \tau_{\psi_2})\}$ be any two SFZNS and $\lambda \geq 0$. Then, the following operation identified as:

- $\mathfrak{S}_{\kappa_1} \supseteq \mathfrak{S}_{\kappa_2} \Leftrightarrow \sigma_2 \geq \sigma_1, \tau_{\sigma_2} \geq \tau_{\sigma_1}, \mu_2 \leq \mu_1, \tau_{\mu_2} \leq \tau_{\mu_1}, \psi_2 \leq \psi_1, \tau_{\psi_2} \leq \tau_{\psi_1}$.
- $\mathfrak{S}_{\kappa_1} = \mathfrak{S}_{\kappa_2} \Leftrightarrow \mathfrak{S}_{\kappa_1} \supseteq \mathfrak{S}_{\kappa_2}$ and $\mathfrak{S}_{\kappa_1} \subseteq \mathfrak{S}_{\kappa_2}$.
- $\mathfrak{S}_{\kappa_1} \cup \mathfrak{S}_{\kappa_2} = \{(\sigma_1 \vee \sigma_2, \tau_{\sigma_1} \vee \tau_{\sigma_2}), (\mu_1 \vee \mu_2, \tau_{\mu_1} \vee \tau_{\mu_2}), (\psi_1 \vee \psi_2, \tau_{\psi_1} \vee \tau_{\psi_2})\}$.
- $\mathfrak{S}_{\kappa_1} \cap \mathfrak{S}_{\kappa_2} = \{(\sigma_1 \wedge \sigma_2, \tau_{\sigma_1} \wedge \tau_{\sigma_2}), (\mu_1 \wedge \mu_2, \tau_{\mu_1} \wedge \tau_{\mu_2}), (\psi_1 \wedge \psi_2, \tau_{\psi_1} \wedge \tau_{\psi_2})\}$.
- $(\mathfrak{S}_{\kappa_1})^c = \{(\sigma_1, \tau_{\sigma_1}), (\mu_1, \tau_{\mu_1}), (\psi_1, \tau_{\psi_1})\}^c = \{(\psi_1, \tau_{\psi_1}), (\mu_1, \tau_{\mu_1}), (\sigma_1, \tau_{\sigma_1})\}$.
- $\mathfrak{S}_{\kappa_1} \oplus \mathfrak{S}_{\kappa_2} = \left\{ \left(\sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1^2 \sigma_2^2}, \sqrt{\tau_{\sigma_1}^2 + \tau_{\sigma_2}^2 - \tau_{\sigma_1}^2 \tau_{\sigma_2}^2} \right), (\mu_1 \mu_2, \tau_{\mu_1} \tau_{\mu_2}), (\psi_1 \psi_2, \tau_{\psi_1} \tau_{\psi_2}) \right\}$.
- $\mathfrak{S}_{\kappa_1} \otimes \mathfrak{S}_{\kappa_2} = \left\{ (\sigma_1 \sigma_2, \tau_{\sigma_1} \tau_{\sigma_2}), (\mu_1 \mu_2, \tau_{\mu_1} \tau_{\mu_2}), \left(\sqrt{\psi_1^2 + \psi_2^2 - \psi_1^2 \psi_2^2}, \sqrt{\tau_{\psi_1}^2 + \tau_{\psi_2}^2 - \tau_{\psi_1}^2 \tau_{\psi_2}^2} \right) \right\}$.
- $\lambda \mathfrak{S}_{\kappa_1} = \left\{ \left(\sqrt{1 - (1 - \sigma_1^2)^\lambda}, \sqrt{1 - (1 - \tau_{\sigma_1}^2)^\lambda} \right), (\mu_1^\lambda, \tau_{\mu_1}^\lambda), (\psi_1^\lambda, \tau_{\psi_1}^\lambda) \right\}$.

$$9. (\mathfrak{S}_{\kappa_1})^\lambda = \left\{ (\sigma_1^\lambda \tau_{\sigma_1}^\lambda), (\mu_1^\lambda \tau_{\mu_1}^\lambda), \left(\sqrt{1 - (1 - \psi_1^2)^\lambda}, \sqrt{1 - (1 - \tau_{\psi_1}^2)^\lambda} \right) \right\}.$$

Definition 6. To compare $SFZN$ $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$, we introduced the score function $\phi(\mathfrak{S}_{\kappa_i}) \in [0, 1]$ as:

$$\phi(\mathfrak{S}_{\kappa_i}) = \frac{2 + (\sigma_{\#}, \tau_{\sigma_{\#}}) - (\mu_{\#}, \tau_{\mu_{\#}}) - (\psi_{\#}, \tau_{\psi_{\#}})}{3} \quad (7)$$

if $\phi(\mathfrak{S}_{\kappa_i}) = \phi(\mathfrak{S}'_{\kappa_i})$, then calculate the accuracy function:

$$\mathfrak{R}(\mathfrak{S}_{\kappa_i}) = \frac{(\sigma_{\#}, \tau_{\sigma_{\#}}) + (\mu_{\#}, \tau_{\mu_{\#}}) + (\psi_{\#}, \tau_{\psi_{\#}})}{3} \quad (8)$$

2.1 $SFZN$ Distance Measure

Definition 7. Suppose $\mathfrak{S}_{\kappa_1} = \{(\sigma_1, \tau_{\sigma_1}), (\mu_1, \tau_{\mu_1}), (\psi_1, \tau_{\psi_1})\}$ and $\mathfrak{S}_{\kappa_2} = \{(\sigma_2, \tau_{\sigma_2}), (\mu_2, \tau_{\mu_2}), (\psi_2, \tau_{\psi_2})\}$ be any two $SFZN$ s. Then, the Euclidean distance between them as follows:

$$d(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}) = \sqrt{\left((\sigma_1 \cdot \tau_{\sigma_1}) - (\sigma_2 \cdot \tau_{\sigma_2})\right)^2 + \left((\mu_1 \cdot \tau_{\mu_1}) - (\mu_2 \cdot \tau_{\mu_2})\right)^2 + \left((\psi_1 \cdot \tau_{\psi_1}) - (\psi_2 \cdot \tau_{\psi_2})\right)^2} \quad (9)$$

2.2 $SFZN$ Entropy Measure

In this part, we introduce a novel entropy metric designed for $SFZN$ derived distance measurements. To achieve this, we adopt the approach presented by Guo and Song [36].

Definition 8. Suppose $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ $i \in N$ be $SFZN$ s. Then, the Entropy measure is identified as follows:

$$\tilde{E}(\mathfrak{S}_{\kappa}) = \frac{1}{n} \sum_{\# = 1}^n \left[\left\{ 1 - d(\mathfrak{S}_{\kappa_i}, \mathfrak{S}_{\kappa_i}^c) \right\} \frac{2 - (\sigma_{\#})^2 \cdot (\tau_{\sigma_{\#}})^2 - (\mu_{\#})^2 \cdot (\tau_{\mu_{\#}})^2 - (\psi_{\#})^2 \cdot (\tau_{\psi_{\#}})^2}{2} \right] \quad (10)$$

Theorem 1. Suppose $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and $\eta_{\kappa_j} = \{(\sigma_j, \tau_{\sigma_j}), (\mu_j, \tau_{\mu_j}), (\psi_j, \tau_{\psi_j})\}$ $i, j = 1, 2, 3, \dots, n$ are $SFZN$ s. Then, the entropy measure of $\tilde{E}(\mathfrak{S}_{\kappa})$ and $\tilde{E}(\eta_{\kappa})$ satisfies the following properties:

1. $\tilde{E}(\mathfrak{S}_{\kappa}) = 0$ iff $\mathfrak{S}_{\kappa}$ is the crisp set.
2. $\tilde{E}(\mathfrak{S}_{\kappa}) \leq \tilde{E}(\eta_{\kappa})$ if $\mathfrak{S}_{\kappa} \leq \eta_{\kappa}$ that is $\sigma_{\#} \leq \sigma_j, \tau_{\sigma_{\#}} \leq \tau_{\sigma_j}, \mu_{\#} \leq \mu_j, \tau_{\mu_{\#}} \leq \tau_{\mu_j}, \psi_{\#} \leq \psi_j$ and $\tau_{\psi_{\#}} \leq \tau_{\psi_j}$
3. $\tilde{E}(\mathfrak{S}_{\kappa}) \leq \tilde{E}(\mathfrak{S}_{\kappa}^c)$

Proof. 1. For a crisp set, we have $\sigma_{\#} = 1, \tau_{\sigma_{\#}} = 1, \mu_{\#} = 0, \tau_{\mu_{\#}} = 0$ and $\psi_{\#} = 0, \tau_{\psi_{\#}} = 0$. Hence, $\tilde{E}(\mathfrak{S}_{\kappa}) = 0$

Conversely, suppose that $\tilde{E}(\mathfrak{S}_{\kappa}) = 0$.

Since $\sigma_{\#}, \tau_{\sigma_{\#}}, \mu_{\#}, \tau_{\mu_{\#}}$ and $\psi_{\#}, \tau_{\psi_{\#}} \in [0, 1]$, $2 - (\sigma_{\#})^2 \cdot (\tau_{\sigma_{\#}})^2 - (\mu_{\#})^2 \cdot (\tau_{\mu_{\#}})^2 - (\psi_{\#})^2 \cdot (\tau_{\psi_{\#}})^2 \neq 0$. Therefore:

$$1 - d(\mathfrak{S}_{\kappa_i}, \mathfrak{S}_{\kappa_i}^c) = 0 \quad (11)$$

only possibility if $\sigma_{\#}, \tau_{\sigma_{\#}} \leq 1$ and $\psi_{\#}, \tau_{\psi_{\#}} \leq 1$. The above equation hold if $\sigma_{\#}, \tau_{\sigma_{\#}} = 1$ and $\psi_{\#}, \tau_{\psi_{\#}} = 1$.

2. Suppose $\mathfrak{S}_{\kappa} \leq \eta_{\kappa}$ then $\mathfrak{S}_{\kappa} \leq \eta_{\kappa}$ that is $\sigma_{\#} \leq \sigma_j, \tau_{\sigma_{\#}} \leq \tau_{\sigma_j}, \mu_{\#} \leq \mu_j, \tau_{\mu_{\#}} \leq \tau_{\mu_j}, \psi_{\#} \leq \psi_j$, and $\tau_{\psi_{\#}} \leq \tau_{\psi_j}$.

For this, we have to show that $\tilde{E}(\eta_\kappa) - \tilde{E}(\mathfrak{S}_\kappa) \geq 0$:

$$\begin{aligned} & \tilde{E}(\eta_\kappa) - \tilde{E}(\mathfrak{S}_\kappa) \\ &= \left\{ \frac{1}{2n} \sum_{\# = 1}^n \left[\left\{ 1 - d(\eta_{\kappa_j}, \eta_{\kappa_j}^c) \right\} 2 - (\sigma_j)^2 \cdot (\tau_{\sigma_j})^2 - (\mu_j)^2 \cdot (\tau_{\mu_j})^2 - (\psi_j)^2 \cdot (\tau_{\psi_j})^2 - \right. \right. \\ & \quad \left. \left. \left\{ 1 - d(\mathfrak{S}_{\kappa_i}, \mathfrak{S}_{\kappa_i}^c) \right\} 2 - (\sigma_\#)^2 \cdot (\tau_{\sigma_\#})^2 - (\mu_\#)^2 \cdot (\tau_{\mu_\#})^2 - (\psi_\#)^2 \cdot (\tau_{\psi_\#})^2 \right] \right\} \\ &= \left\{ \frac{1}{2n} \sum_{\# = 1}^n \left[\left\{ ((\sigma_j)^2 - (\sigma_\#)^2) \cdot ((\tau_{\sigma_j})^2 - (\tau_{\sigma_\#})^2) + ((\mu_j)^2 - (\mu_\#)^2) \cdot ((\tau_{\mu_j})^2 - (\tau_{\mu_\#})^2) \right. \right. \\ & \quad + 3((\psi_j)^2 - (\psi_\#)^2) \cdot ((\tau_{\psi_j})^2 - (\tau_{\psi_\#})^2) + ((\sigma_\#)^2 (\mu_\#)^2 - (\sigma_j)^2 (\mu_j)^2) \cdot ((\tau_{\sigma_\#})^2 \\ & \quad (\tau_{\mu_\#})^2 - (\tau_{\sigma_j})^2 (\tau_{\mu_j})^2) + ((\mu_j)^2 (\psi_j)^2 - (\mu_\#)^2 (\psi_\#)^2) \cdot ((\tau_{\mu_j})^2 (\tau_{\psi_j})^2 - (\tau_{\mu_\#})^2 (\tau_{\psi_\#})^2) \\ & \quad \left. \left. + ((\sigma_\#)^4 - (\sigma_j)^4) \cdot ((\tau_{\sigma_\#})^4 - (\tau_{\sigma_j})^4) + ((\psi_j)^4 - (\psi_\#)^4) \cdot ((\tau_{\psi_j})^4 - (\tau_{\psi_\#})^4) \right\} \right] \right\} \geq 0. \end{aligned}$$

Since, all power are even, then it follows that:

$$\tilde{E}(\mathfrak{S}_\kappa) \leq \tilde{E}(\eta_\kappa) \quad (12)$$

3. Since we have:

$$\begin{aligned} \tilde{E}(\mathfrak{S}_\kappa) &= \frac{1}{n} \sum_{\# = 1}^n \left[\left\{ 1 - d(\mathfrak{S}_{\kappa_i}, \mathfrak{S}_{\kappa_i}^c) \right\} \frac{2 - (\sigma_\#)^2 \cdot (\tau_{\sigma_\#})^2 - (\mu_\#)^2 \cdot (\tau_{\mu_\#})^2 - (\psi_\#)^2 \cdot (\tau_{\psi_\#})^2}{2} \right] \\ &= \frac{1}{n} \sum_{\# = 1}^n \left[\left\{ 1 - d(\mathfrak{S}_{\kappa_i}^c, \mathfrak{S}_{\kappa_i}) \right\} \frac{2 - (\psi_\#)^2 \cdot (\tau_{\psi_\#})^2 - (\mu_\#)^2 \cdot (\tau_{\mu_\#})^2 - (\sigma_\#)^2 \cdot (\tau_{\sigma_\#})^2}{2} \right] \\ &= \tilde{E}(\mathfrak{S}_\kappa^c) \end{aligned}$$

□

Definition 9. [37] The t-norm and t-conorm of AA operator is identified as:

$$T_{AA}^\wp(i, \ell) = \begin{cases} T_{\tilde{D}}(i, \ell), & \text{if } \wp = 0, \\ \min(i, \ell), & \text{if } \wp = +\infty, \\ e^{-((- \ln i)^\wp + (- \ln \ell)^\wp)^{\frac{1}{\wp}}}, & \text{if } 0 < \wp < +\infty \end{cases}$$

where $T_{\tilde{D}}(i, \ell)$ denote the drastic t-norms and $T_{\tilde{P}}(i, \ell)$ denote the algebraic product.

$$S_{AA}^\wp(i, \ell) = \begin{cases} S_{\tilde{D}}(i, \ell), & \text{if } \wp = 0, \\ \max(i, \ell), & \text{if } \wp = +\infty, \\ 1 - e^{-((- \ln(1-i))^\wp + (- \ln(1-\ell))^\wp)^{\frac{1}{\wp}}}, & \text{if } 0 < \wp < +\infty, \end{cases}$$

where $S_{\tilde{D}}(i, \ell)$ denote the drastic t-conorm and $S_{\tilde{P}}(i, \ell)$ denote the algebraic sum.

3. Aczel-Alsina Aggregation Operators based on Spherical Fuzzy Z-number

This section explores many basic processes as well as the underlying ideas of the AA aggregation operation. We start by outlining the special qualities and benefits of the AA aggregation and outlining its theoretical underpinnings. Next, we provide fundamental procedures that are the core of our methodology, demonstrating their smooth integration with the aggregation process. Aim is to present a clear and comprehensive framework that bolsters the novel parts of our study by going into great detail about these ideas.

Definition 10. Let $\mathfrak{S}_{\kappa_1} = \{(\sigma_1, \tau_{\sigma_1}), (\mu_1, \tau_{\mu_1}), (\psi_1, \tau_{\psi_1})\}$ and $\mathfrak{S}_{\kappa_2} = \{(\sigma_2, \tau_{\sigma_2}), (\mu_2, \tau_{\mu_2}), (\gamma_{S_2}, \tau_{\psi_2})\}$ be any two SFZNs. Then, some basic operation of SFZNs based on AA are given as:

$$\begin{aligned}
 (1) \ \mathfrak{S}_{\kappa_1} \oplus \mathfrak{S}_{\kappa_2} &= \left\{ \begin{array}{l} \left(\sqrt{1 - e^{-((- \ln(1-(\sigma_1)^2))^{\wp} + (- \ln(1-(\sigma_2)^2))^{\wp})^{\frac{1}{\wp}}}}, \right. \\ \left. \sqrt{1 - e^{-((- \ln(1-(\tau_{\sigma_1})^2))^{\wp} + (- \ln(1-(\tau_{\sigma_2})^2))^{\wp})^{\frac{1}{\wp}}}} \right) \\ \left(\sqrt{e^{-((- \ln(\mu_1)^2))^{\wp} + (- \ln(\mu_2)^2))^{\wp})^{\frac{1}{\wp}}}}, \sqrt{e^{-((- \ln(\tau_{\mu_1})^2))^{\wp} + (- \ln(\tau_{\mu_2})^2))^{\wp})^{\frac{1}{\wp}}}} \right) \\ \left(\sqrt{e^{-((- \ln(\psi_1)^2))^{\wp} + (- \ln(\psi_2)^2))^{\wp})^{\frac{1}{\wp}}}}, \sqrt{e^{-((- \ln(\tau_{\psi_1})^2))^{\wp} + (- \ln(\tau_{\psi_2})^2))^{\wp})^{\frac{1}{\wp}}}} \right) \end{array} \right\} \\
 (2) \ \mathfrak{S}_{\kappa_1} \otimes \mathfrak{S}_{\kappa_2} &= \left\{ \begin{array}{l} \left(\sqrt{e^{-((- \ln(\sigma_1)^2))^{\wp} + (- \ln(\sigma_2)^2))^{\wp})^{\frac{1}{\wp}}}}, \sqrt{e^{-((- \ln(\tau_{\sigma_1})^2))^{\wp} + (- \ln(\tau_{\sigma_2})^2))^{\wp})^{\frac{1}{\wp}}}} \right) \\ \left(\sqrt{e^{-((- \ln(\mu_1)^2))^{\wp} + (- \ln(\mu_2)^2))^{\wp})^{\frac{1}{\wp}}}}, \sqrt{e^{-((- \ln(\tau_{\mu_1})^2))^{\wp} + (- \ln(\tau_{\mu_2})^2))^{\wp})^{\frac{1}{\wp}}}} \right) \\ \left(\sqrt{1 - e^{-((- \ln(1-(\psi_1)^2))^{\wp} + (- \ln(1-(\psi_2)^2))^{\wp})^{\frac{1}{\wp}}}}, \right. \\ \left. \sqrt{1 - e^{-((- \ln(1-(\tau_{\psi_1})^2))^{\wp} + (- \ln(1-(\tau_{\psi_2})^2))^{\wp})^{\frac{1}{\wp}}}} \right) \end{array} \right\} \\
 (3) \ \Omega \odot \mathfrak{S}_{\kappa_1} &= \left\{ \begin{array}{l} \left(\sqrt{1 - e^{-((\Omega(- \ln(1-(\sigma_1)^2))^{\wp})^{\frac{1}{\wp}})}, \sqrt{1 - e^{-((\Omega(- \ln(1-(\tau_{\sigma_1})^2))^{\wp})^{\frac{1}{\wp}}}} \right) \\ \left(\sqrt{e^{-((\Omega(- \ln(\mu_1)^2))^{\wp})^{\frac{1}{\wp}}}}, \sqrt{e^{-((\Omega(- \ln(\tau_{\mu_1})^2))^{\wp})^{\frac{1}{\wp}}}} \right) \\ \left(\sqrt{e^{-((\Omega(- \ln(\psi_1)^2))^{\wp})^{\frac{1}{\wp}}}}, \sqrt{e^{-((\Omega(- \ln(\tau_{\psi_1})^2))^{\wp})^{\frac{1}{\wp}}}} \right) \end{array} \right\} \\
 (4) \ \mathfrak{S}_{\kappa_1}^{\Omega} &= \left\{ \begin{array}{l} \left(\sqrt{e^{-((\Omega(- \ln(\sigma_1)^2))^{\wp})^{\frac{1}{\wp}}}}, \sqrt{e^{-((\Omega(- \ln(\tau_{\sigma_1})^2))^{\wp})^{\frac{1}{\wp}}}} \right) \\ \left(\sqrt{e^{-((\Omega(- \ln(\mu_1)^2))^{\wp})^{\frac{1}{\wp}}}}, \sqrt{e^{-((\Omega(- \ln(\tau_{\mu_1})^2))^{\wp})^{\frac{1}{\wp}}}} \right) \\ \left(\sqrt{1 - e^{-((\Omega(- \ln(1-(\psi_1)^2))^{\wp})^{\frac{1}{\wp}})}, \sqrt{1 - e^{-((\Omega(- \ln(1-(\tau_{\psi_1})^2))^{\wp})^{\frac{1}{\wp}}}} \right) \end{array} \right\}
 \end{aligned}$$

4. Aggregation Operators of *SFZN* by using Aczel-Alsina Aggregation Operator

Formulating the average and geometric aggregation operators based on AA t-norms (T-N) and t-conorms (T-CN) is the main goal of this section. Proofs of these aggregation operators will be thoroughly established, guaranteeing their theoretical soundness. Furthermore, we shall investigate their basic traits and provide a comprehensive study of their behavior.

4.1 Spherical Fuzzy Z-number Weighted Arithmetic Aczel-Alsina Operator

In the framework of the *SFZN* environment, we introduce the *SFZNNWA_{AA}*, *SFZNOWA_{AA}*, and *SFZNHWA_{AA}* operators in this subsection. These novel operators aim to enhance the aggregation process and provide more flexible and nuanced decision-making by including spherical fuzzy Z-number. We will first provide a full explanation of these operators before delving into their special qualities and practical applications.

Definition 11. Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\sharp}, \tau_{\sigma_{\sharp}}), (\mu_{\sharp}, \tau_{\mu_{\sharp}}), (\psi_{\sharp}, \tau_{\psi_{\sharp}})\}$ and $\sharp \in \varrho N$ a clustering of *SFZN* and *SFZNNWA_{AA}*: $SFZN^{\wp} \rightarrow SFZN$ is identified as:

$$SFZNNWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \sum_{\sharp=1}^n \Omega_{\sharp} \mathfrak{S}_{\kappa_{\sharp}} \quad (13)$$

where $\sum_{\sharp=1}^n \Omega_{\sharp} = 1$ and $\Omega_{\sharp} \in [0, 1]$.

Theorem 2. Let $\mathfrak{S}_{\kappa_{\sharp}} = \{(\sigma_{\sharp}, \tau_{\sigma_{\sharp}}), (\mu_{\sharp}, \tau_{\mu_{\sharp}}), (\psi_{\sharp}, \tau_{\psi_{\sharp}})\}$ the clustering of *SFZN*s, where $\sharp \in \varrho N$. Then, *SFZNNWA_{AA}* is identified as:

$$SFZNOWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \sum_{\# = 1}^n \Omega_{\#} \mathfrak{S}_{\kappa_{\#}} \quad (14)$$

$$= \left\{ \begin{array}{l} \left(\sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(1 - (\sigma_{\#})^2))^{\varphi})^{\frac{1}{\varphi}}}}, \sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(1 - (\tau_{\sigma_{\#}})^2))^{\varphi})^{\frac{1}{\varphi}}}} \right) \\ \left(\sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\mu_{\#})^2))^{\varphi})^{\frac{1}{\varphi}}}}, \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\tau_{\mu_{\#}})^2))^{\varphi})^{\frac{1}{\varphi}}}} \right) \\ \left(\sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\psi_{\#})^2))^{\varphi})^{\frac{1}{\varphi}}}}, \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\tau_{\psi_{\#}})^2))^{\varphi})^{\frac{1}{\varphi}}}} \right) \end{array} \right\}.$$

Proof. Appendix contains the Theorem's proof. $\square$

Property 1. Idempotency: Let $\mathfrak{S}_{\kappa} = \mathfrak{S}_{\kappa_i} = \{(\sigma, \tau_{\sigma}), (\mu, \tau_{\mu}), (\psi, \tau_{\psi})\}$ and $\# \in \varrho N$ are a clustering of $SFZN$ if all the $\mathfrak{S}_{\kappa_i}$ are identical. Then:

$$SFZNOWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \mathfrak{S}_{\kappa} \quad (15)$$

Proof. Appendix provides proof of this Property. $\square$

Property 2. Monotonicity: Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and

$\mathfrak{S}_{\kappa_{i'}} = \{(\sigma_{\#'}, \tau_{\sigma_{\#'}}), (\mu_{\#'}, \tau_{\mu_{\#'}}), (\psi_{\#'}, \tau_{\psi_{\#'}})\}$ $i, \# \in \varrho N$ be a clustering of $SFZN$ such that $\mathfrak{S}_{\kappa_i} \subseteq \mathfrak{S}_{\kappa_{i'}}$. Then:

$$SFZNOWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) \leq SFZNOWA_{AA}(\mathfrak{S}_{\kappa'_1}, \mathfrak{S}_{\kappa'_2}, \mathfrak{S}_{\kappa'_3}, \dots, \mathfrak{S}_{\kappa'_n}) \quad (16)$$

Proof. Appendix provides the proof of this Property. $\square$

Property 3. Boundedness: Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and $\# \in \varrho N$ be a clustering of $SFZN$ such that $\mathfrak{S}_{\kappa_L} = \max_{\#} \mathfrak{S}_{\kappa_i}$ and $\mathfrak{S}_{\kappa_j} = \min_{\#} \mathfrak{S}_{\kappa_i}$. Then:

$$\mathfrak{S}_{\kappa_j} \leq SFZNOWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) \leq \mathfrak{S}_{\kappa_L} \quad (17)$$

Proof. Proof is basic. $\square$

Definition 12. Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and $\# \in \varrho N$ are a clustering of $SFZN$ and $SFZNOWA_{AA} \cdot SFZN^{\varphi} \rightarrow SFZN$ is identified as:

$$SFZNOWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \sum_{\# = 1}^n \Omega_{\#} \mathfrak{S}_{\kappa_{\delta(\#)}} \quad (18)$$

where $\sum_{\# = 1}^n \Omega_{\#} = 1$ and $\Omega_{\#} \in [0, 1]$.

Theorem 3. Let $\mathfrak{S}_{\kappa_{\#}} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ the clustering of $SFZN$ s, where $\# \in \varrho N$ then $SFZNOWA_{AA}$ is identified as:

$$SFZNOWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \sum_{\# = 1}^n \Omega_{\#} \mathfrak{S}_{\kappa_{\delta(\#)}} \quad (19)$$

$$= \left\{ \begin{array}{l} \left(\sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(1 - (\sigma_{\delta(\#)}^2)^{\varphi}))^{\frac{1}{\varphi}})}, \sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(1 - (\tau_{\sigma_{\delta(\#)}^2)^{\varphi}))^{\frac{1}{\varphi}})} \right)} \\ \left(\sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\mu_{\delta(\#)}^2)^{\varphi})^{\frac{1}{\varphi}})}, \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\tau_{\mu_{\delta(\#)}^2)^{\varphi})^{\frac{1}{\varphi}})} \right)} \\ \left(\sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\psi_{\delta(\#)}^2)^{\varphi})^{\frac{1}{\varphi}})}, \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\tau_{\psi_{\delta(\#)}^2)^{\varphi})^{\frac{1}{\varphi}})} \right) \end{array} \right\}$$

Property 4. Idempotency: Let $\mathfrak{S}_{\kappa} = \mathfrak{S}_{\kappa_i} = \{(\sigma, \tau_{\sigma}), (\mu, \tau_{\mu}), (\psi, \tau_{\psi})\}$ and $\# \in \varrho N$ are a clustering of $SFZN$. If all the $\mathfrak{S}_{\kappa_i}$ are identical, then:

$$SFZNOWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \mathfrak{S}_{\kappa} \quad (20)$$

Property 5. Monotonicity: Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and

$\mathfrak{S}_{\kappa_{i'}} = \{(\sigma_{\#'}, \tau_{\sigma_{\#'}}), (\mu_{\#'}, \tau_{\mu_{\#'}}), (\psi_{\#'}, \tau_{\psi_{\#'}})\}$ $i, \# \in \varrho N$ be a clustering of $SFZN$ such that $\mathfrak{S}_{\kappa_i} \subseteq \mathfrak{S}_{\kappa_{i'}}$. Then:

$$SFZNOWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) \leq SFZNOWA_{AA}(\mathfrak{S}_{\kappa'_1}, \mathfrak{S}_{\kappa'_2}, \mathfrak{S}_{\kappa'_3}, \dots, \mathfrak{S}_{\kappa'_n}) \quad (21)$$

Property 6. Boundedness: Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and $\# \in \varrho N$ be a clustering of $SFZN$ such that $\mathfrak{S}_{\kappa_L} = \max_{\#} \mathfrak{S}_{\kappa_i}$ and $\mathfrak{S}_{\kappa_j} = \min_{\#} \mathfrak{S}_{\kappa_i}$. Then:

$$\mathfrak{S}_{\kappa_j} \leq SFZNOWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) \leq \mathfrak{S}_{\kappa_L} \quad (22)$$

Definition 13. Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and $\# \in \varrho N$ are a clustering of $SFZN$ and $SFZNHWA_{AA}: SFZN^{\varphi} \rightarrow SFZN$ is identified as:

$$SFZNHWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \sum_{\# = 1}^n \Omega_i \mathfrak{S}'_{\kappa_{\delta(\#)}} \quad (23)$$

with n dimension such that i -th highest weighted value is $\mathfrak{S}_{\kappa_{\delta(\#)}}, \mathfrak{S}'_{\kappa_{\delta(\#)}} = n\lambda_{\#} \mathfrak{S}_{\kappa_{\#}} (\# \in N)$, where $0 \leq \lambda_{\#} \leq 1$ and $\sum_{\# = 1}^n \lambda_{\#} = 1$, while $\lambda_{\#}$ is the associated weight vector.

Theorem 4. Let $\mathfrak{S}_{\kappa_{\#}} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ is the clustering of $SFZN$ s, where $\# \in \varrho N$. Then, $SFZNHWA_{AA}$ is identified as:

$$SFZNHWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \sum_{\# = 1}^n \lambda_i \mathfrak{S}'_{\kappa_{\delta(\#)}} \quad (24)$$

$$= \left\{ \begin{array}{l} \left(\sqrt{1 - e^{-(\sum_{\# = 1}^n \lambda_{\#} (-\ln(1 - (\sigma'_{\delta(\#)}^2)^{\varphi}))^{\frac{1}{\varphi}})}, \sqrt{1 - e^{-(\sum_{\# = 1}^n \lambda_{\#} (-\ln(1 - (\tau'_{\sigma_{\delta(\#)}^2)^{\varphi}))^{\frac{1}{\varphi}})} \right)} \\ \left(\sqrt{e^{-(\sum_{\# = 1}^n \lambda_{\#} (-\ln(\mu'_{\delta(\#)}^2)^{\varphi})^{\frac{1}{\varphi}})}, \sqrt{e^{-(\sum_{\# = 1}^n \lambda_{\#} (-\ln(\tau'_{\mu_{\delta(\#)}^2)^{\varphi})^{\frac{1}{\varphi}})} \right)} \\ \left(\sqrt{e^{-(\sum_{\# = 1}^n \lambda_{\#} (-\ln(\psi'_{\delta(\#)}^2)^{\varphi})^{\frac{1}{\varphi}})}, \sqrt{e^{-(\sum_{\# = 1}^n \lambda_{\#} (-\ln(\tau'_{\psi_{\delta(\#)}^2)^{\varphi})^{\frac{1}{\varphi}})} \right) \end{array} \right\}.$$

Property 7. Idempotency: Let $\mathfrak{S}_{\kappa} = \mathfrak{S}_{\kappa_i} = \{(\sigma, \tau_{\sigma}), (\mu, \tau_{\mu}), (\psi, \tau_{\psi})\}$ and $\# \in \varrho N$ are a clustering of $SFZN$ if all the $\mathfrak{S}_{\kappa_i}$ are identical. Then:

$$SFZNHWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \mathfrak{S}_{\kappa} \quad (25)$$

Property 8. Monotonicity: Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and

$\mathfrak{S}_{\kappa_{\#'}} = \{(\sigma_{\#'}, \tau_{\sigma_{\#'}}), (\mu_{\#'}, \tau_{\mu_{\#'}}), (\psi_{\#'}, \tau_{\psi_{\#'}})\}$ $i, \# \in \varrho N$ be a clustering of $SFZN$ such that $\mathfrak{S}_{\kappa_i} \subseteq \mathfrak{S}_{\kappa'_i}$. Then:

$$SFZNHWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) \leq SFZNHWA_{AA}(\mathfrak{S}_{\kappa'_1}, \mathfrak{S}_{\kappa'_2}, \mathfrak{S}_{\kappa'_3}, \dots, \mathfrak{S}_{\kappa'_n}) \quad (26)$$

Property 9. Boundedness: Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and $\# \in \varrho N$ be a clustering of $SFZN$ such that $\mathfrak{S}_{\kappa_L} = \max_{\#} \mathfrak{S}_{\kappa_i}$ and $\mathfrak{S}_{\kappa_j} = \min_{\#} \mathfrak{S}_{\kappa_i}$. Then:

$$\mathfrak{S}_{\kappa_j} \leq SFZNHWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) \leq \mathfrak{S}_{\kappa_L} \quad (27)$$

4.2 Spherical Fuzzy Z-number Weighted Geometric Aczel-Alsina Operator

A variety of functions have been used to enact a variety of fuzzy instances. These all-inclusive membership function types are used for both the aggregation stage and the more elastic decision-making process using geometric fuzzy Z-number. In the context of the $SFZN$ environment, this part introduces the operators $SFZNWG_{AA}$, $SFZNOWG_{AA}$, and $SFZNHWG_{AA}$. These creative operators use spherical fuzzy Z-number to improve the aggregation process and provide more nuanced and flexible decision-making. Before examining these operators' particular properties and uses, their thorough explanation will be provided.

Definition 14. Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and $\# \in \varrho N$ are a clustering of $SFZN$ and $SFZNWG_{AA}: SFZN^{\varphi} \rightarrow SFZN$ is identified as:

$$SFZNWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \sum_{\#=1}^n \mathfrak{S}_{\kappa_{\#}}^{\Omega_{\#}} \quad (28)$$

where $\sum_{\#=1}^n \Omega_{\#} = 1$ and $\Omega_{\#} \in [0, 1]$.

Theorem 5. Let $\mathfrak{S}_{\kappa_{\#}} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ be the clustering of $SFZN$ s, where $\# \in \varrho N$. Then, $SFZNWG_{AA}$ is identified as:

$$SFZNWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \sum_{\#=1}^n \mathfrak{S}_{\kappa_{\#}}^{\Omega_{\#}} \quad (29)$$

$$= \left\{ \begin{array}{l} \left(\sqrt[n]{e^{-(\sum_{\#=1}^n \Omega_{\#} (-\ln(\sigma_{\#})^2)^{\varphi})^{\frac{1}{\varphi}}}}, \sqrt[n]{e^{-(\sum_{\#=1}^n \Omega_{\#} (-\ln(\tau_{\sigma_{\#}})^2)^{\varphi})^{\frac{1}{\varphi}}}} \right) \\ \left(\sqrt[n]{e^{-(\sum_{\#=1}^n \Omega_{\#} (-\ln(\mu_{\#})^2)^{\varphi})^{\frac{1}{\varphi}}}}, \sqrt[n]{e^{-(\sum_{\#=1}^n \Omega_{\#} (-\ln(\tau_{\mu_{\#}})^2)^{\varphi})^{\frac{1}{\varphi}}}} \right) \\ \left(\sqrt[n]{1 - e^{-(\sum_{\#=1}^n \Omega_{\#} (-\ln(1-(\psi_{\#})^2)^{\varphi})^{\frac{1}{\varphi}}}}, \sqrt[n]{1 - e^{-(\sum_{\#=1}^n \Omega_{\#} (-\ln(1-(\tau_{\psi_{\#}})^2)^{\varphi})^{\frac{1}{\varphi}}}} \right) \end{array} \right\}.$$

Property 10. Idempotency: Let $\mathfrak{S}_{\kappa} = \mathfrak{S}_{\kappa_i} = \{(\sigma, \tau_{\sigma}), (\mu, \tau_{\mu}), (\psi, \tau_{\psi})\}$ and $\# \in \varrho N$ are a clustering of $SFZN$ if all the $\mathfrak{S}_{\kappa_i}$ are identical. Then:

$$SFZNWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \mathfrak{S}_{\kappa} \quad (30)$$

Property 11. Monotonicity: Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and

$\mathfrak{S}_{\kappa_{\#'}} = \{(\sigma_{\#'}, \tau_{\sigma_{\#'}}), (\mu_{\#'}, \tau_{\mu_{\#'}}), (\psi_{\#'}, \tau_{\psi_{\#'}})\}$ $i, \# \in \varrho N$ be a clustering of $SFZN$ such that $\mathfrak{S}_{\kappa_i} \subseteq \mathfrak{S}_{\kappa'_i}$. Then:

$$SFZNWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) \leq SFZNWG_{AA}(\mathfrak{S}_{\kappa'_1}, \mathfrak{S}_{\kappa'_2}, \mathfrak{S}_{\kappa'_3}, \dots, \mathfrak{S}_{\kappa'_n}) \quad (31)$$

Property 12. Boundedness: Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and $\# \in \varrho N$ be a clustering of $SFZN$ such that $\mathfrak{S}_{\kappa_L} = \max_{\#} \mathfrak{S}_{\kappa_i}$ and $\mathfrak{S}_{\kappa_j} = \min_{\#} \mathfrak{S}_{\kappa_i}$. Then:

$$\mathfrak{S}_{\kappa_j} \leq SFZNOWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) \leq \mathfrak{S}_{\kappa_L} \quad (32)$$

Proof. Proof is straight forward. $\square$

Definition 15. Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and $\# \in \varrho N$ be a clustering of $SFZN$ and $SFZNOWG_{AA}: SFZN^{\varphi} \rightarrow SFZN$ is identified as:

$$SFZNOWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \sum_{\#=1}^n \mathfrak{S}_{\kappa_{\delta(\#)}}^{\Omega_{\#}} \quad (33)$$

where $\sum_{\#=1}^n \Omega_{\#} = 1$ and $\Omega_{\#} \in [0, 1]$.

Theorem 6. Let $\mathfrak{S}_{\kappa_{\#}} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ be the clustering of $SFZN$ s, where $\# \in \varrho N$. Then, $SFZNOWG_{AA}$ is identified as:

$$SFZNOWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \sum_{\#=1}^n \mathfrak{S}_{\kappa_{\delta(\#)}}^{\Omega_{\#}} \quad (34)$$

$$= \left\{ \begin{array}{l} \left(\sqrt[n]{e^{-(\sum_{\#=1}^n \Omega_{\#} (-\ln(\sigma_{\delta(\#)}))^2)^{\varphi}}}, \sqrt[n]{e^{-(\sum_{\#=1}^n \Omega_{\#} (-\ln(\tau_{\sigma_{\delta(\#)}}))^2)^{\varphi}}} \right) \\ \left(\sqrt[n]{e^{-(\sum_{\#=1}^n \Omega_{\#} (-\ln(\mu_{\delta(\#)}))^2)^{\varphi}}}, \sqrt[n]{e^{-(\sum_{\#=1}^n \Omega_{\#} (-\ln(\tau_{\mu_{\delta(\#)}}))^2)^{\varphi}}} \right) \\ \left(\sqrt[n]{1 - e^{-(\sum_{\#=1}^n \Omega_{\#} (-\ln(1 - (\psi_{\delta(\#)}))^2)^{\varphi})}}, \sqrt[n]{1 - e^{-(\sum_{\#=1}^n \Omega_{\#} (-\ln(1 - (\tau_{\psi_{\delta(\#)}}))^2)^{\varphi})}} \right) \end{array} \right\}$$

Property 13. Idempotency: Let $\mathfrak{S}_{\kappa} = \mathfrak{S}_{\kappa_i} = \{(\sigma, \tau_{\sigma}), (\mu, \tau_{\mu}), (\psi, \tau_{\psi})\}$ and $\# \in \varrho N$ are a clustering of $SFZN$ if all the $\mathfrak{S}_{\kappa_i}$ are identical. Then:

$$SFZNOWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \mathfrak{S}_{\kappa} \quad (35)$$

Property 14. Monotonicity: Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and $\mathfrak{S}_{\kappa_{\#'}} = \{(\sigma_{\#'}, \tau_{\sigma_{\#'}}), (\mu_{\#'}, \tau_{\mu_{\#'}}), (\psi_{\#'}, \tau_{\psi_{\#'}})\}$ $i, \# \in \varrho N$ be a clustering of $SFZN$ such that $\mathfrak{S}_{\kappa_i} \subseteq \mathfrak{S}_{\kappa_{\#'}}'$. Then:

$$SFZNOWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) \leq SFZNOWG_{AA}(\mathfrak{S}_{\kappa_1'}, \mathfrak{S}_{\kappa_2'}, \mathfrak{S}_{\kappa_3'}, \dots, \mathfrak{S}_{\kappa_n'}) \quad (36)$$

Property 15. Boundedness: Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and $\# \in \varrho N$ be a clustering of $SFZN$ such that $\mathfrak{S}_{\kappa_L} = \max_{\#} \mathfrak{S}_{\kappa_i}$ and $\mathfrak{S}_{\kappa_j} = \min_{\#} \mathfrak{S}_{\kappa_i}$. Then:

$$\mathfrak{S}_{\kappa_j} \leq SFZNOWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) \leq \mathfrak{S}_{\kappa_L} \quad (37)$$

Definition 16. Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and $\# \in \varrho N$ are a clustering of $SFZN$ and $SFZNHWG_{AA}: SFZN^{\varphi} \rightarrow SFZN$ is identified as:

$$SFZNHWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \sum_{\#=1}^n (\mathfrak{S}_{\kappa_{\delta(\#)}}')^{\Omega_{\#}} \quad (38)$$

with n dimension such that i -th highest weighted value is $\mathfrak{S}_{\kappa_{\delta(\#)}}$ and $\mathfrak{S}'_{\kappa_{\delta(\#)}} = n\lambda_{\#}\mathfrak{S}_{\kappa_{\#}}$ ($\# \in N$), where $0 \leq \lambda_{\#} \leq 1$ and $\sum_{\#}^n \lambda_{\#} = 1$, while $\lambda_{\#}$ is the associated weight vector.

Theorem 7. Let $\mathfrak{S}_{\kappa_{\#}} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ be the clustering of $SFZN$ s, where $\# \in \varrho N$. Then, $SFZNHWG_{AA}$ is identified as:

$$SFZNHWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \sum_{\#}^n (\mathfrak{S}'_{\kappa_{\delta(\#)}})^{\Omega_{\#}} \quad (39)$$

$$= \left\{ \begin{array}{l} \left(\sqrt[n]{e^{-(\sum_{\#}^n \lambda_{\#} (-\ln(\sigma'_{\delta(\#)}))^2)^{\frac{1}{\wp}}}}, \sqrt[n]{e^{-(\sum_{\#}^n \lambda_{\#} (-\ln(\tau'_{\sigma'_{\delta(\#)}}))^2)^{\frac{1}{\wp}}}} \right) \\ \left(\sqrt[n]{e^{-(\sum_{\#}^n \lambda_{\#} (-\ln(\mu'_{\delta(\#)}))^2)^{\frac{1}{\wp}}}}, \sqrt[n]{e^{-(\sum_{\#}^n \lambda_{\#} (-\ln(\tau'_{\mu'_{\delta(\#)}}))^2)^{\frac{1}{\wp}}}} \right) \\ \left(\sqrt[n]{1 - e^{-(\sum_{\#}^n \lambda_{\#} (-\ln(1 - (\psi'_{\delta(\#)}))^2)^{\frac{1}{\wp}}}}}, \sqrt[n]{1 - e^{-(\sum_{\#}^n \lambda_{\#} (-\ln(1 - (\tau'_{\psi'_{\delta(\#)}}))^2)^{\frac{1}{\wp}}}} \right) \end{array} \right\}.$$

Property 16. Idempotency: Let $\mathfrak{S}_{\kappa} = \mathfrak{S}_{\kappa_i} = \{(\sigma, \tau_{\sigma}), (\mu, \tau_{\mu}), (\psi, \tau_{\psi})\}$ and $\# \in \varrho N$ are a clustering of $SFZN$ if all the $\mathfrak{S}_{\kappa_i}$ are identical. Then:

$$SFZNHWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \mathfrak{S}_{\kappa} \quad (40)$$

Property 17. Monotonicity: Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and $\mathfrak{S}_{\kappa_{i'}} = \{(\sigma_{\#'}, \tau_{\sigma_{\#'}}), (\mu_{\#'}, \tau_{\mu_{\#'}}), (\psi_{\#'}, \tau_{\psi_{\#'}})\}$ $i, \# \in \varrho N$ be a clustering of $SFZN$ such that $\mathfrak{S}_{\kappa_i} \subseteq \mathfrak{S}_{\kappa_{i'}}$. Then:

$$SFZNHWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) \leq SFZNHWG_{AA}(\mathfrak{S}_{\kappa'_1}, \mathfrak{S}_{\kappa'_2}, \mathfrak{S}_{\kappa'_3}, \dots, \mathfrak{S}_{\kappa'_n}) \quad (41)$$

Property 18. Boundedness: Let $\mathfrak{S}_{\kappa_i} = \{(\sigma_{\#}, \tau_{\sigma_{\#}}), (\mu_{\#}, \tau_{\mu_{\#}}), (\psi_{\#}, \tau_{\psi_{\#}})\}$ and $\# \in \varrho N$ be a clustering of $SFZN$ such that $\mathfrak{S}_{\kappa_L} = \max_{\#} \mathfrak{S}_{\kappa_i}$ and $\mathfrak{S}_{\kappa_j} = \min_{\#} \mathfrak{S}_{\kappa_i}$. Then:

$$\mathfrak{S}_{\kappa_j} \leq SFZNHWG_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) \leq \mathfrak{S}_{\kappa_L} \quad (42)$$

5. $SFZN_{AA}$ Operator for Multi-Attributes Group Decision-Making

Within this segment, an algorithmic approach is devised to address the MAGDM dilemma by employing the geometric and average aggregation operators that were suggested. In addition, a MAGDM example was presented to demonstrate the practical implementation of these operators.

Assume an alternative set denoted by $\$ = \{\$, \$2, \$3, \dots, \$n\}$ and an attribute set denoted by $\aleph = \{\aleph_1, \aleph_2, \aleph_3, \dots, \aleph_n\}$, each accompanied by its corresponding weight vector $\omega = \{\omega_1, \omega_2, \dots, \omega_n\}$. To ensure a valid weighting scheme, the weight vectors must adhere to the conditions that they are members of a closed unit interval (i.e., range from 0 to 1) and their sum must be equal to 1. Consider a spherical fuzzy Z -number decision matrix is presented as $\mathfrak{S}_{\kappa}^k = [\mathfrak{S}_{\kappa_{ij}}^k]_{m \times n}$.

Algorithm

- Step 1. Expert generate decision matrices.
- Step 2. Positive and negative criteria are terms that are frequently applied in the MAGDM procedure. When evaluating a negative criterion, denoted as "cost", its value is transformed into "benefit" through the complement operation.

Step 3. Calculate experts weights.

Step 3a. The idea of the perfect solution based on group decision-making (GDIS) is very similar to the personal views held by every decision maker (DM). Therefore, finding out the best GDIS requires finding an average of the individual DM opinions. In this phase, GDIS is calculated using spherical fuzzy Z -number weighted mean of the alternative values that are connected to the criteria provided by the DMs. This calculation considers an equal weighting of the DM values shown below:

$$GDIS = \begin{pmatrix} IS_{11} & IS_{12} & IS_{13} & \cdot & \cdot & IS_{1n} \\ IS_{21} & IS_{22} & IS_{23} & \cdot & \cdot & IS_{2n} \\ IS_{31} & IS_{32} & IS_{33} & \cdot & \cdot & IS_{3n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ IS_{m1} & IS_{m2} & IS_{m3} & \cdot & \cdot & IS_{mn} \end{pmatrix}$$

where:

$$IS_{ij} = \sum_{k=1}^m \frac{1}{m} N_{ij}^k \quad (43)$$

$$= \left\{ \begin{array}{c} \left(\sqrt[1/\phi]{1 - e^{-(\sum_{\# = 1}^n \frac{1}{m} (-\ln(1 - (\sigma_{\#}^2)))^{\phi}})}, \right. \\ \left. \sqrt[1/\phi]{1 - e^{-(\sum_{\# = 1}^n \frac{1}{m} (-\ln(1 - (\tau_{\sigma_{\#}}^2)))^{\phi}})} \right) \\ \left(\sqrt[1/\phi]{e^{-(\sum_{\# = 1}^n \frac{1}{m} (-\ln(\mu_{\#}^2))^{\phi})}}, \sqrt[1/\phi]{e^{-(\sum_{\# = 1}^n \frac{1}{m} (-\ln(\tau_{\mu_{\#}}^2))^{\phi})}} \right) \\ \left(\sqrt[1/\phi]{e^{-(\sum_{\# = 1}^n \frac{1}{m} (-\ln(\psi_{\#}^2))^{\phi})}}, \sqrt[1/\phi]{e^{-(\sum_{\# = 1}^n \frac{1}{m} (-\ln(\tau_{\psi_{\#}}^2))^{\phi})}} \right) \end{array} \right\}$$

Step 3b. To obtain the group right ideal decision (GRID) and group left ideal decision (GLID), the following computations were done:

$$GRID = \begin{pmatrix} RID_{11} & RID_{12} & RID_{13} & \cdot & \cdot & RID_{1n} \\ RID_{21} & RID_{22} & RID_{23} & \cdot & \cdot & RID_{2n} \\ RID_{31} & RID_{32} & RID_{33} & \cdot & \cdot & RID_{3n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ RID_{m1} & RID_{m2} & RID_{m3} & \cdot & \cdot & RID_{mn} \end{pmatrix}$$

where :

$$RID_{ij} = \left\{ (N_{ij}^k) : \max_k [sc(N_{ij}^k)] \right\} \quad (44)$$

and

$$GLID = \begin{pmatrix} LID_{11} & LID_{12} & LID_{13} & \cdot & \cdot & LID_{1n} \\ LID_{21} & LID_{22} & LID_{23} & \cdot & \cdot & LID_{2n} \\ LID_{31} & LID_{32} & LID_{33} & \cdot & \cdot & LID_{3n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ LID_{m1} & LID_{m2} & LID_{m3} & \cdot & \cdot & LID_{mn} \end{pmatrix}$$

where:

$$LID_{ij} = \left\{ (N_{ij}^k) : \min_k [sc(N_{ij}^k)] \right\} \quad (45)$$

Step 3c. The distance of decision matrix N_{ij}^k to GDIS, GRID, and GLID are computed in this

step using Definition 7. The distances are denoted symbolically as DGDIS, DGRID, and DGLID, respectively.

Step 3d. In this particular stage, we compute the indices of how close things are (CIs) in the following manner:

$$CI^{(k)} = \frac{\sum_{\# = 1}^n DGRID_{\#}^{(k)} + \sum_{\# = 1}^n DGLID_{\#}^{(k)}}{\sum_{\# = 1}^n DGDIS_{\#}^{(k)} + \sum_{\# = 1}^n DGRID_{\#}^{(k)} + \sum_{\# = 1}^n DGLID_{\#}^{(k)}} \quad (46)$$

Step 3e. This is how weights for DMs are calculated in this step:

$$Y^{(k)} = \frac{CI^{(k)}}{\sum_{\# = 1}^m CI^{(k)}}$$

Step 4. The weight of the used attribute has been worked out using the suggested SF entropy measure

Step 4a. Determine as follows the revised group decision (RGDIS):

$$RIS_{ij} = \sum_{k=1}^m Y^{(k)} N_{ij}^k \quad (47)$$

$$= \left\{ \begin{array}{l} \left(\sqrt[1/\varphi]{1 - e^{-(\sum_{\# = 1}^n Y^{(k)} (-\ln(1 - (\sigma_{\#}^2)))^{\varphi})}}, \sqrt[1/\varphi]{1 - e^{-(\sum_{\# = 1}^n Y^{(k)} (-\ln(1 - (\tau_{\sigma_{\#}}^2)))^{\varphi})}} \right) \\ \left(\sqrt[1/\varphi]{e^{-(\sum_{\# = 1}^n Y^{(k)} (-\ln(\mu_{\#}^2))^{\varphi})}}, \sqrt[1/\varphi]{e^{-(\sum_{\# = 1}^n Y^{(k)} (-\ln(\tau_{\mu_{\#}}^2))^{\varphi})}} \right) \\ \left(\sqrt[1/\varphi]{e^{-(\sum_{\# = 1}^n Y^{(k)} (-\ln(\psi_{\#}^2))^{\varphi})}}, \sqrt[1/\varphi]{e^{-(\sum_{\# = 1}^n Y^{(k)} (-\ln(\tau_{\psi_{\#}}^2))^{\varphi})}} \right) \end{array} \right\}.$$

Step 4b. Using Definition 8, *SFZN* entropy measure corresponding to each attribute is computed as follows:

$$EA_j = \tilde{E}(RIS_{1j}, RIS_{2j}, RIS_{3j}, \dots, RIS_{nj}) \quad (48)$$

Step 4c. Attribute weights are calculate as follows:

$$\Theta A_j = \frac{1 - EA_j}{o - \sum_{j=1}^o EA_j} \quad (49)$$

Step 5. Find the matrix which is a collective spherical fuzzy *z*-number decision matrix $\mathfrak{S}_{\kappa} = [\mathfrak{S}_{\kappa_{ij}}]_{m \times n}$.

Step 6. The MAGDM process involves integrating the attributes for each alternative using the arithmetic and geometric aggregation operators for spherical fuzzy *z*-number AA.

Step 7. Calculated score values for each alternative according to the scoring Definition.

Step 8. All alternatives are ranked in descending order and the best one is chosen.

The flow chart of the algorithm is illustrated in Figure 1.

6. Numerical Problem

Our decision-making environment deals with real-time quality control in conjunction with logistics tracking, wherein a plethora of alternatives are assessed by the desired performance indicators. The data under consideration for this analysis is simulated so that it embodies realistic conditions, while still obeying the structure of *SFZN*. The decision matrix consists of four alternatives IoT (Internet of Things)-enabled tracking devices, block chain-based tracking, RFID technology and collaborative inventory, all representing a different quality control and logistics strategy. Each alternative is evaluated according to four pertinent attributes; i.e. real-time tracking, data accuracy and reliability, cost-effectiveness, as well as security and privacy. Attributes were chosen for consideration because of their importance in creating an optimized and efficient supply chain process.

The evaluation of each single attribute is carried out by suitable experts, making use of SFZNs,

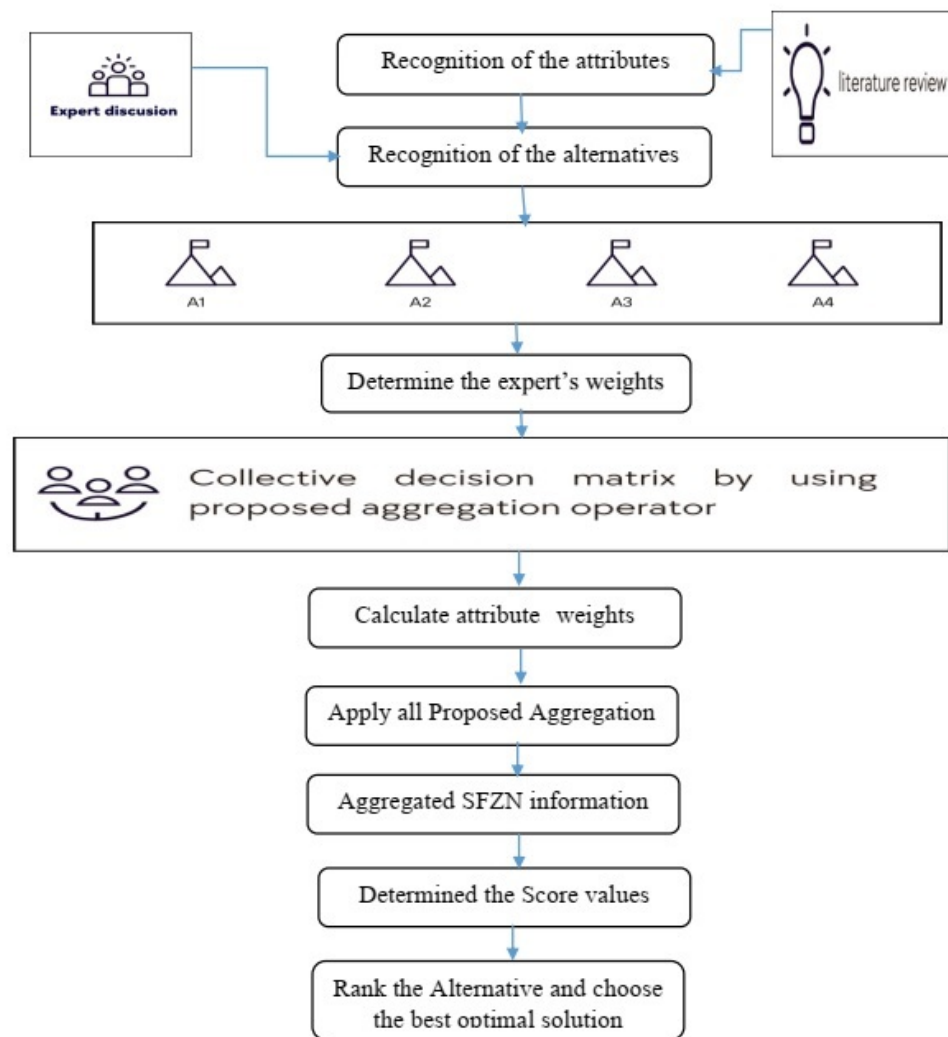


Fig. 1. Flow chart of the algorithm

with the membership, non-membership, neutrality, and reliability measures assigned in order to capture the inherent uncertainty and the decision-maker's levels of confidence. Also, with attributes like logistics efficiency, together with operational costs, these attributes are quantified using the SFZNs so as to not miss any aspect of evaluation under uncertainty. All decision matrices are shown in the case study section, with the proposed spherical fuzzy Aczel-Alsina AOs for ranking the alternatives. This structured approach increases precision to the decision-making with commendable strength in a real-time logistics and quality control environment.

The primary objectives of the problem are improved visibility into the supply chain process, reduce transportation costs and improve delivery times. In the realm of optimizing supply chains, employing group decision-making is pivotal for addressing the inherent challenges posed by uncertain and imprecise supply chain data. In today's dynamic and intricate landscape of supply chain management, the imperative for real-time logistics tracking has risen to paramount importance.

This is a report on a real-life occurrence where a credible and globally acknowledged distributor and manufacturer of consumer electronics, availed itself of innovative technological approaches and collaborative decision-making strategies in order to enhance its supply chain through real-time logistics tracking. A company that had been facing numerous supply chain challenges such as unending delays, stock discrepancies as well as rising transport costs chose to embrace transformation. They intend to implement the utilization of real-time logistics tracking technologies so as to address these

issues. For this journey, advanced technology among others like GPS tracking, IoT-based sensors, complex data analysis tools which can harmoniously blend with their supply chain operations were very costly for them to buy.

Data security concerns, difficulties merging data silos, and change management are just some of the challenges that affect these transformation stories. The company's future plans include applications for predictive analytics and machine learning, increased use of blockchain technology for improved traceability, as well as expanding collaborative platforms into the supply chain network involving other firms. To wrap it up, this case study provides its readers with evidence and insight into what can be achieved by working together towards a common objective, good decision making ability, and a constant stream of information on transportation related matters. It also proposes recommendations for these firms that desire to augment their supply chains efficiency and brand themselves globally in ever changing marketplaces. Moreover, blockchain as a way of improving tracking, machine learning for predictive analysis and a cooperative platform that would allow international partners to interface with supply chain management systems are some of the possible commercial initiatives we will look into. This motivates professionals who are concerned about collaborative decision-making and real-time monitoring of logistics. In this fast pacing world market, these strategies might help businesses prosper.

Real-time logistics monitoring provides transport companies with timely and precise data about shipments locations, storage status and routes taken by the freight. This enables them to allocate resources appropriately allocate resources and make adjustments in transit corridors quickly in order to cope with changing customer demands leading to rapidly adapting supply chains or technologies for scarce goods thus avoiding possible emergencies in surrounding economies. It helps them respond quickly to unexpected disturbances and reduce harmful effects on business activities and profit margins. Organizations can lower expenditures considerably while improving productivity and customer satisfaction by incorporating real-time data into transportation planning, inventory control, and resource deployment.

One international distribution company aims to enhance the efficiency of its supply chain through implementing real-time logistics monitoring systems. Their key objectives are visibility enhancement, operational cost reduction, and timely delivery assurance. Monitoring tools facilitate business management in this case by enabling them to deal with the uncertainties associated with supply chain management through collective decision-making involving many people, analyzing risks, adapting to shifting situations and creating a collaborative atmosphere for employees. Real-time monitoring and the complicated area of fuzzy group decision-making meet in such a way that they form an enduring foundation for achieving efficiency in volatile and unpredictable supply chains.

There are four attributes:

- **Real-time tracking** ($\aleph_1$) - Ability to provide accurate real-time tracking of shipments and their conditions. Degree of granularity in data collection (location, temperature, humidity, etc.).
- **Data accuracy and reliability** ($\aleph_2$) - Precision of data collected and transmitted by the tracking solution. Dependability of data accuracy over varying environmental conditions.
- **Cost-effectiveness** ($\aleph_3$) - Initial investment cost for implementation. Operational expenses, including data transmission and device maintenance.
- **Security and privacy** ($\aleph_4$) - Measures in place to secure data transmission and storage. Protection of sensitive information from unauthorized access.

There are four alternatives:

- **IoT-enabled tracking devices** ($\$1$) - Attach IoT tracking devices to shipments for real-time data on location, temperature, humidity, and more. Utilize cellular networks for data transmission.
- **Block chain-based tracking** ($\$2$) - Implement a blockchain platform to create an immutable and transparent record of all supply chain events. Enhance security and trust in the supply chain by preventing data tampering.
- **RFID technology** ($\$3$) - Utilize RFID tags to track and identify shipments automatically. Improve

scanning efficiency and accuracy in warehouses and during transit.

- **Collaborative inventory** ($\$4$) - Exploring partnerships or collaborations with other organizations to share inventory and resources, reducing costs and improving efficiency.

7. Results and discussions

This section provides the experimental results of the studied problem, a comparison with existing techniques to show its validity, and a discussion of the results.

7.1 Experimental results

This study has established three professional decision matrices on testing optimal means of improving real time quality control in a dynamic environment of logistics tracking. A three-expert domain decision-making team was recruited, as it is stipulated in Table 3. The chosen experts are both industrial and academic professionals who can be viewed as experienced specialists in the field of supply chain analytics, management of logistics, and data security. Second, a sequence of informative on-line meetings was made to introduce the real-time logistics tracking issue, its operational objectives, and the assessment system framework under the *SFZN* environment. What was being looked at in these sessions was the necessity of ensuring clarity and involving the experts in the matter. Third, the evaluations of the four alternatives to be applied on the four identified criteria were given by the experts through a well-designed questionnaire whereby reliable and uncertain information could be gathered which were needed in the analysis.

Table 3
Details about the experts

Expert	Experience (Years)	Gender	Occupation	Position
D_1	22	Male	Supply Chain Analyst	Academia
D_2	15	Female	Logistics Operations Lead	Industry
D_3	18	Male	Data Security Specialist	Industry

The alternatives within the decision-making framework amount to four options, so-written as $\$ = \{\$1, \$2, \$3, \$4\}$, which indicate IoT-enabled tracking devices, blockchain-based tracking, RFID technology, and collaborative inventory systems. These candidates are evaluated with respect to four critical criteria denoted by $\aleph = \{\aleph_1, \aleph_2, \aleph_3, \aleph_4\}$. To provide proportionate emphasis on practical significance a weight vector of $\lambda = 0.15, 0.25, 0.25, 0.35$ was allocated. The opinions of three experts in the sphere of supply chain analytics, industrial systems engineering, and information security were gathered. In *SFZN* environment, the experts independently presented their assessment and the evaluations modeled into membership, non-membership, neutrality, and reliability measures. The synthetic data used is based on the simulation of the realistic working conditions of a real-life situation of a worldwide distributor of electronics and its desire to decrease the time delays, increase the visibility of the stock of all kinds and cut the transportation costs by employing digital transformation. The computations of the matrices gathered are what configures the consideration of using spherical fuzzy AczelAlsina aggregation operators to rank as well as recommend the most feasible solution.

Step 1. Decision matrices created by the expert 1, expert 2, and expert 3 are presented in Table 4.

Step 2. Normalization was unnecessary in light of the benefit-oriented criterion design.

Step 3. Expert weights were calculated based on the process defined in Step 3 of the algorithm. Its application is used in this method to determine the significant levels for each of the experts. For simplicity, the intermediate calculations are omitted here, while the final weights are presented as $Y^1 = 0.3259$, $Y^2 = 0.3321$, and $Y^3 = 0.3420$. This method incorporates the significance to each expert. These weights also imply the observed dominance of certain individuals over others in the decision-making process.

Step 4. Entropy measures were used in the calculation of weights of the attributes, as proposed in

Step 4 of our algorithm. This method allowed for a quantitative assessment of the individual attributes added value since variability in the decision matrix was considered. The intermediate calculation tables are omitted in this document but the final attribute weights of the keys are presented as $EA_1 = 0.148$, $EA_2 = 0.279$, $EA_3 = 0.292$, and $EA_4 = 0.280$. These weights are actually an indication of the amount or degree of influence that each of the attributes contributes towards a decision.

Table 4
Expert's decision matrices

$\mathfrak{S}_{\kappa_{ij}}$		$\aleph_1$	$\aleph_2$
D_1	$\$1$	((0.3, 0.4), (0.5, 0.6), (0.4, 0.3))	((0.6, 0.4), (0.3, 0.4), (0.5, 0.4))
	$\$2$	((0.7, 0.3), (0.6, 0.4), (0.1, 0.3))	((0.5, 0.2), (0.2, 0.3), (0.3, 0.3))
	$\$3$	((0.3, 0.1), (0.2, 0.6), (0.5, 0.4))	((0.4, 0.1), (0.7, 0.2), (0.4, 0.2))
	$\$4$	((0.4, 0.3), (0.5, 0.2), (0.1, 0.6))	((0.3, 0.7), (0.1, 0.1), (0.2, 0.6))
$\mathfrak{S}_{\kappa_{ij}}$		$\aleph_1$	$\aleph_2$
D_2	$\$1$	((0.2, 0.4), (0.3, 0.6), (0.3, 0.3))	((0.3, 0.4), (0.3, 0.3), (0.3, 0.3))
	$\$2$	((0.3, 0.2), (0.4, 0.7), (0.2, 0.4))	((0.2, 0.3), (0.2, 0.4), (0.2, 0.2))
	$\$3$	((0.5, 0.3), (0.5, 0.2), (0.4, 0.1))	((0.4, 0.2), (0.4, 0.1), (0.4, 0.5))
	$\$4$	((0.1, 0.1), (0.6, 0.3), (0.2, 0.5))	((0.3, 0.1), (0.1, 0.2), (0.5, 0.4))
$\mathfrak{S}_{\kappa_{ij}}$		$\aleph_1$	$\aleph_2$
D_3	$\$1$	((0.1, 0.3), (0.5, 0.2), (0.3, 0.2))	((0.4, 0.3), (0.1, 0.4), (0.1, 0.4))
	$\$2$	((0.2, 0.2), (0.4, 0.4), (0.5, 0.4))	((0.2, 0.1), (0.2, 0.3), (0.3, 0.3))
	$\$3$	((0.4, 0.1), (0.2, 0.1), (0.4, 0.3))	((0.4, 0.2), (0.4, 0.2), (0.4, 0.3))
	$\$4$	((0.3, 0.1), (0.3, 0.2), (0.1, 0.5))	((0.3, 0.4), (0.3, 0.1), (0.2, 0.2))
$\mathfrak{S}_{\kappa_{ij}}$		$\aleph_3$	$\aleph_4$
D_1	$\$1$	((0.4, 0.8), (0.2, 0.3), (0.1, 0.3))	((0.2, 0.1), (0.6, 0.4), (0.6, 0.3))
	$\$2$	((0.2, 0.2), (0.3, 0.2), (0.2, 0.5))	((0.4, 0.5), (0.3, 0.3), (0.4, 0.3))
	$\$3$	((0.1, 0.1), (0.4, 0.1), (0.3, 0.6))	((0.6, 0.3), (0.2, 0.5), (0.6, 0.4))
	$\$4$	((0.6, 0.3), (0.5, 0.5), (0.4, 0.4))	((0.1, 0.2), (0.4, 0.2), (0.3, 0.6))
$\mathfrak{S}_{\kappa_{ij}}$		$\aleph_3$	$\aleph_4$
D_2	$\$1$	((0.1, 0.3), (0.1, 0.1), (0.2, 0.3))	((0.4, 0.3), (0.3, 0.1), (0.3, 0.2))
	$\$2$	((0.2, 0.5), (0.5, 0.2), (0.4, 0.5))	((0.5, 0.2), (0.2, 0.3), (0.4, 0.4))
	$\$3$	((0.3, 0.3), (0.3, 0.3), (0.3, 0.6))	((0.3, 0.4), (0.5, 0.4), (0.2, 0.5))
	$\$4$	((0.4, 0.2), (0.2, 0.4), (0.4, 0.4))	((0.2, 0.1), (0.3, 0.2), (0.1, 0.3))
$\mathfrak{S}_{\kappa_{ij}}$		$\aleph_3$	$\aleph_4$
D_3	$\$1$	((0.4, 0.1), (0.5, 0.2), (0.4, 0.3))	((0.2, 0.3), (0.4, 0.2), (0.4, 0.4))
	$\$2$	((0.2, 0.2), (0.3, 0.3), (0.3, 0.1))	((0.5, 0.2), (0.3, 0.4), (0.3, 0.3))
	$\$3$	((0.1, 0.3), (0.2, 0.1), (0.2, 0.3))	((0.3, 0.1), (0.2, 0.3), (0.1, 0.1))
	$\$4$	((0.4, 0.4), (0.4, 0.4), (0.4, 0.4))	((0.1, 0.2), (0.1, 0.2), (0.3, 0.4))

Step 5. The collective decision matrix was calculated in Table 5.

Table 5
Collective decision matrix

$\mathfrak{S}_{\kappa_{ij}}$	$\aleph_1$
$\$1$	((0.21589, 0.36971), (0.42198, 0.41207), (0.32949, 0.26115))
$\$2$	((0.48227, 0.23789), (0.45651, 0.48170), (0.21828, 0.36420))

Table 5
Continued

$\mathfrak{S}_{\kappa_{ij}}$	$\aleph_1$
$\$3$	$((0.41188, 0.19318), (0.27114, 0.22572), (0.43017, 0.22876))$
$\$4$	$((0.29711, 0.19187), (0.44606, 0.22883), (0.12588, 0.53061))$
$\mathfrak{S}_{\kappa_{ij}}$	$\aleph_2$
$\$1$	$((0.45884, 0.36971), (0.20604, 0.36355), (0.24336, 0.36355))$
$\$2$	$((0.33795, 0.21662), (0.20000, 0.33008), (0.26221, 0.26221))$
$\$3$	$((0.40000, 0.17414), (0.48003, 0.15888), (0.40000, 0.31146))$
$\$4$	$((0.30000, 0.49602), (0.14560, 0.12588), (0.27114, 0.36016))$
$\mathfrak{S}_{\kappa_{ij}}$	$\aleph_3$
$\$1$	$((0.33599, 0.55469), (0.21735, 0.18132), (0.20224, 0.30000))$
$\$2$	$((0.20000, 0.33995), (0.35547, 0.22975), (0.28922, 0.28835))$
$\$3$	$((0.19318, 0.25429), (0.28682, 0.14403), (0.26115, 0.47337))$
$\$4$	$((0.48087, 0.31417), (0.34172, 0.43017), (0.40000, 0.40000))$
$\mathfrak{S}_{\kappa_{ij}}$	$\aleph_4$
$\$1$	$((0.28573, 0.25429), (0.41491, 0.19914), (0.41491, 0.28931))$
$\$2$	$((0.47094, 0.33795), (0.26221, 0.33102), (0.36252, 0.33008))$
$\$3$	$((0.43430, 0.29659), (0.27114, 0.38986), (0.22572, 0.26813))$
$\$4$	$((0.14165, 0.17361), (0.22629, 0.20000), (0.20829, 0.41491))$

Step 6. The attributes were incorporated into all alternatives using the $SFZNWA_{AA}$, $SFZNG_{AA}$, $SFZNOW_{AA}$, $SFZNG_{AA}$, $SFZNHW_{AA}$ and $SFZNHWG_{AA}$ operators, as shown in Table 6.

Step 7. The scoring definition were utilized to compute the score values of every alternative:

Table 6

Usage of the $SFZNWA_{AA}$, $SFZNG_{AA}$, $SFZNOW_{AA}$, $SFZNG_{AA}$, $SFZNHW_{AA}$ and $SFZNHWG_{AA}$ operators

$SFZNWA_{AA}$	
$\mathfrak{S}_{\kappa_{\#}}$	$\aleph_j$
$\$1$	$((0.35146, 0.41484), (0.28354, 0.25565), (0.28038, 0.30733))$
$\$2$	$((0.37909, 0.29623), (0.28883, 0.31462), (0.28792, 0.30227))$
$\$3$	$((0.36832, 0.24039), (0.32367, 0.20948), (0.30443, 0.32281))$
$\$4$	$((0.33949, 0.34054), (0.24990, 0.22460), (0.25216, 0.40960))$
$SFZNG_{AA}$	
$\mathfrak{S}_{\kappa_{\#}}$	$\aleph_j$
$\$1$	$((0.32842, 0.37518), (0.28354, 0.25565), (0.30721, 0.31143))$
$\$2$	$((0.33574, 0.28419), (0.28883, 0.31462), (0.29656, 0.30639))$
$\$3$	$((0.33269, 0.22966), (0.32367, 0.20948), (0.32851, 0.35090))$
$\$4$	$((0.27900, 0.28131), (0.24990, 0.22460), (0.28865, 0.41748))$
$SFZNOW_{AA}$	
$\mathfrak{S}_{\kappa_{\#}}$	$\aleph_j$
$\$1$	$((0.33879, 0.38017), (0.31190, 0.28523), (0.30163, 0.30162))$
$\$2$	$((0.38116, 0.28308), (0.31076, 0.33059), (0.26927, 0.30622))$

Table 6
Continued

$SFZNOWA_{AA}$	
$\mathfrak{S}_3$	$((0.36470, 0.22590), (0.32385, 0.19483), (0.33146, 0.31621))$
$\mathfrak{S}_4$	$((0.33658, 0.29398), (0.29050, 0.24109), (0.22480, 0.43250))$
$SFZNOWG_{AA}$	
$\mathfrak{S}_{\kappa_{\mathfrak{g}}}$	$\mathfrak{N}_j$
$\mathfrak{S}_1$	$((0.30919, 0.35230), (0.31190, 0.28523), (0.32352, 0.30674))$
$\mathfrak{S}_2$	$((0.33680, 0.27132), (0.31076, 0.33059), (0.27712, 0.31160))$
$\mathfrak{S}_3$	$((0.33036, 0.21701), (0.32385, 0.19483), (0.35447, 0.34750))$
$\mathfrak{S}_4$	$((0.27725, 0.24696), (0.29050, 0.24109), (0.27250, 0.44316))$
$SFZNHWA_{AA}$	
$\mathfrak{S}_{\kappa_{\mathfrak{g}}}$	$\mathfrak{N}_j$
$\mathfrak{S}_1$	$((0.33717, 0.37848), (0.32199, 0.29612), (0.31643, 0.32670))$
$\mathfrak{S}_2$	$((0.36471, 0.28011), (0.32032, 0.34102), (0.29478, 0.32236))$
$\mathfrak{S}_3$	$((0.35337, 0.22396), (0.34676, 0.21316), (0.34214, 0.34280))$
$\mathfrak{S}_4$	$((0.33282, 0.29947), (0.29540, 0.25938), (0.25802, 0.44214))$
$SFZNHWG_{AA}$	
$\mathfrak{S}_{\kappa_{\mathfrak{g}}}$	$\mathfrak{N}_j$
$\mathfrak{S}_1$	$((0.34067, 0.36020), (0.32199, 0.29612), (0.31003, 0.30133))$
$\mathfrak{S}_2$	$((0.34985, 0.34221), (0.32032, 0.34102), (0.27589, 0.29966))$
$\mathfrak{S}_3$	$((0.31655, 0.18376), (0.32217, 0.18632), (0.33830, 0.36097))$
$\mathfrak{S}_4$	$((0.29297, 0.32267), (0.27334, 0.25649), (0.30678, 0.42225))$

- **By using $SFZNWA_{AA}$:** $\phi(\$1) = 0.66480$, $\phi(\$2) = 0.66238$, $\phi(\$3) = 0.64082$, and $\phi(\$4) = 0.65206$.
- **By using $SFZNWG_{AA}$:** $\phi(\$1) = 0.65168$, $\phi(\$2) = 0.63789$, $\phi(\$3) = 0.63111$, and $\phi(\$4) = 0.63395$.
- **By using $SFZNWA_{AA}$:** $\phi(\$1) = 0.64962$, $\phi(\$2) = 0.64090$, $\phi(\$3) = 0.63816$, and $\phi(\$4) = 0.64389$.
- **By using $SFZNWG_{AA}$:** $\phi(\$1) = 0.64024$, $\phi(\$2) = 0.63410$, $\phi(\$3) = 0.62847$, and $\phi(\$4) = 0.62589$.
- **By using $SFZNHWA_{AA}$:** $\phi(\$1) = 0.64269$, $\phi(\$2) = 0.63263$, $\phi(\$3) = 0.62931$, and $\phi(\$4) = 0.63632$.
- **By using $SFZNHWG_{AA}$:** $\phi(\$1) = 0.64465$, $\phi(\$2) = 0.64260$, $\phi(\$3) = 0.62534$, and $\phi(\$4) = 0.63163$.

Step 8. Table 7 presents the order all options from highest to lowest.

Table 7
Numerical problem ranking

Sr.	Operators	Scoring
1	$SFZNWA_{AA}$	$\phi(\$1) \geq \phi(\$2) \geq \phi(\$4) \geq \phi(\$3)$
2	$SFZNWG_{AA}$	$\phi(\$1) \geq \phi(\$2) \geq \phi(\$4) \geq \phi(\$3)$
3	$SFZNWA_{AA}$	$\phi(\$1) \geq \phi(\$4) \geq \phi(\$2) \geq \phi(\$3)$
4	$SFZNWG_{AA}$	$\phi(\$1) \geq \phi(\$2) \geq \phi(\$3) \geq \phi(\$4)$
5	$SFZNHWA_{AA}$	$\phi(\$1) \geq \phi(\$4) \geq \phi(\$2) \geq \phi(\$3)$
6	$SFZNHWG_{AA}$	$\phi(\$1) \geq \phi(\$2) \geq \phi(\$4) \geq \phi(\$3)$

Consequently, it was determined that the option S_1 represents the optimal course of action. All operators (including $SFZNWA_{AA}$, $SFZNWG_{AA}$, $SFZNOWA_{AA}$, $SFZNOWG_{AA}$, $SFZNHWA_{AA}$, and $SFZNHWG_{AA}$) are graphically represented in Figure 2.

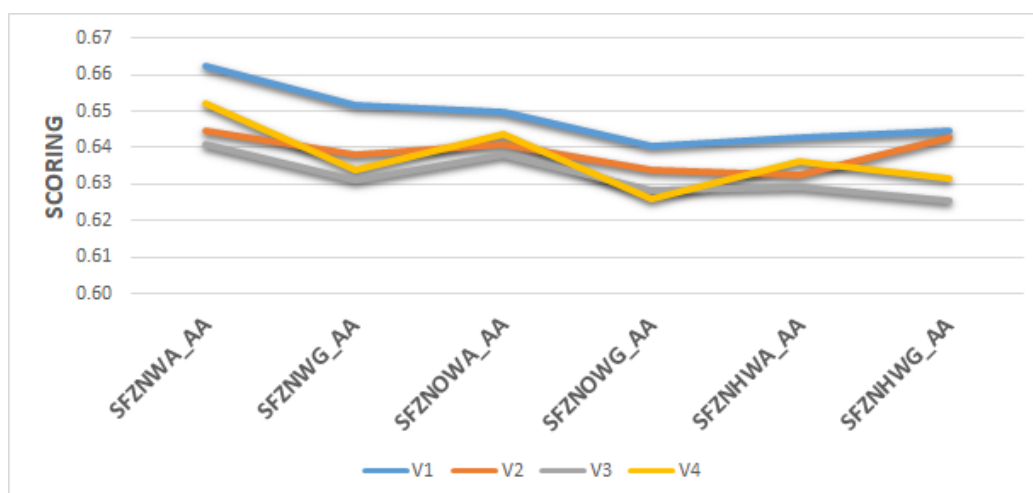


Fig. 2. Graphical representation of the rankings

When considering the application of MAGDM to the issue of real-time logistics tracking for supply chain optimization, we conducted a thorough evaluation of the effectiveness of each logistics strategy in optimizing supply chain operations using four distinct criteria and four logistics strategies as potential alternatives. The aforementioned criteria, which may comprise supply chain visibility, inventory management efficiency, shipment delivery time, and cost reduction, form the basis for evaluating the effect of each logistics strategy on supply chain performance. Upon performing an exhaustive examination and comparison of the selected criteria for each strategy, it becomes indisputable that the implementation of a real-time monitoring and Internet of Things integration strategy substantially surpasses alternative approaches in terms of augmenting supply chain efficiency, diminishing expenses, and enhancing visibility.

The research results underscore the substantial impact that MAGDM methodologies have on the classification of diverse logistics strategies when various decision criteria are utilized. Updated MAGDM method proposed here is an ultimate advance because it uses hybrid evaluation data which includes spherical values and spherical measures for determining mass dependability. In contrast, conventional spherical MAGDM techniques are solely reliant on univariate evaluation data, thus ignoring the varying nature of dependability indices during decision-making processes. This shows that real-time logistics tracking can have huge impacts on supply chain optimization and decision-making processes to accommodate higher productivity levels while ensuring competition in today's changing business environment.

7.2 Comparison Analysis

This part compares the created MADM technique with AA aggregation operators that are proposed therein, highlighting the advantages of the suggested method. On the other hand, attributes of the various aggregation operators presented in this work are compared to those of $SFZN$ operators suggested by Ashraf [24]. Thus, we show how practical decision-making problems (DMPs) can effectively be resolved amid uncertainty using the method, for which reason we assert its consistency and strength.

Step 1. The decision matrix, as presented in Table 8 by the experts [24].

Step 2. Since the criteria are benefit-type, there is no need for the normalization.

Step 3. Because it is a decision matrix of collective spherical fuzzy z -number, it is not necessary to aggregate it.

Step 4. Incorporate attributes in all alternatives by means of the $SFZNWA_{AA}$, $SFZNWG_{AA}$,

Table 8
Decision matrix by the experts [24]

$\mathfrak{S}_{\kappa_{ij}}$	$\aleph_1$	$\aleph_2$	$\aleph_3$
$\$1$	((0.6, 0.2), (0.3, 0.4), (0.4, 0.5))	((0.7, 0.4), (0.4, 0.4), (0.2, 0.3))	((0.4, 0.4), (0.2, 0.6), (0.2, 0.4))
$\$2$	((0.4, 0.3), (0.3, 0.5), (0.2, 0.3))	((0.6, 0.7), (0.1, 0.3), (0.2, 0.5))	((0.4, 0.5), (0.4, 0.6), (0.2, 0.4))
$\$3$	((0.6, 0.3), (0.2, 0.6), (0.2, 0.4))	((0.7, 0.3), (0.2, 0.5), (0.3, 0.4))	((0.2, 0.5), (0.3, 0.4), (0.6, 0.3))
$\$4$	((0.7, 0.3), (0.4, 0.6), (0.5, 0.4))	((0.6, 0.4), (0.3, 0.4), (0.4, 0.6))	((0.4, 0.3), (0.2, 0.7), (0.3, 0.5))

$SFZNOWA_{AA}$, $SFZNOWG_{AA}$, $SFZNHWA_{AA}$ and $SFZNHWG_{AA}$ operators, as shown in Table 9.

Table 9

Usage of the $SFZNWA_{AA}$, $SFZNWG_{AA}$, $SFZNOWA_{AA}$, $SFZNOWG_{AA}$, $SFZNHWA_{AA}$, and $SFZNHWG_{AA}$ operators

$SFZNWA_{AA}$	
$\mathfrak{S}_{\kappa_{\#}}$	$\aleph_j$
$\$1$	((0.62327, 0.32495), (0.31491, 0.42508), (0.27702, 0.39823))
$\$2$	((0.49241, 0.53908), (0.20633, 0.42320), (0.20000, 0.38033))
$\$3$	((0.61326, 0.34061), (0.21254, 0.52680), (0.27511, 0.38311))
$\$4$	((0.63311, 0.34244), (0.32317, 0.52635), (0.42547, 0.48252))
$SFZNWG_{AA}$	
$\mathfrak{S}_{\kappa_{\#}}$	$\aleph_j$
$\$1$	((0.59866, 0.28879), (0.31491, 0.42508), (0.31368, 0.42305))
$\$2$	((0.46663, 0.44692), (0.20633, 0.42320), (0.20000, 0.40566))
$\$3$	((0.53954, 0.32389), (0.21254, 0.52680), (0.33886, 0.38711))
$\$4$	((0.60702, 0.33466), (0.32317, 0.52635), (0.44040, 0.50521))
$SFZNOWA_{AA}$	
$\mathfrak{S}_{\kappa_{\#}}$	$\aleph_j$
$\$1$	((0.60180, 0.37816), (0.29439, 0.46663), (0.22191, 0.36131))
$\$2$	((0.51067, 0.57152), (0.18690, 0.40418), (0.20000, 0.39823))
$\$3$	((0.62347, 0.34061), (0.21254, 0.51823), (0.28534, 0.38311))
$\$4$	((0.56044, 0.35157), (0.26850, 0.52581), (0.37078, 0.52680))
$SFZNOWG_{AA}$	
$\mathfrak{S}_{\kappa_{\#}}$	$\aleph_j$
$\$1$	((0.55297, 0.36050), (0.29439, 0.46663), (0.24298, 0.37749))
$\$2$	((0.48397, 0.48233), (0.18690, 0.40418), (0.20000, 0.42305))
$\$3$	((0.54708, 0.32389), (0.21254, 0.51823), (0.34507, 0.38711))
$\$4$	((0.52635, 0.34343), (0.26850, 0.52581), (0.38556, 0.54032))
$SFZNHWA_{AA}$	
$\mathfrak{S}_{\kappa_{\#}}$	$\aleph_j$
$\$1$	((0.66042, 0.39355), (0.24319, 0.38140), (0.17953, 0.30357))
$\$2$	((0.55126, 0.62890), (0.13987, 0.34703), (0.14542, 0.34669))
$\$3$	((0.64090, 0.40255), (0.16624, 0.42801), (0.26266, 0.30342))
$\$4$	((0.63393, 0.38244), (0.22566, 0.45394), (0.32471, 0.45140))

Table 9
Continued

$SFZNHWG_{AA}$	
$\mathfrak{S}_{\mathcal{R}_\#}$	$\mathfrak{N}_j$
$\$1$	$((0.48380, 0.15646), (0.19384, 0.30044), (0.41845, 0.51440))$
$\$2$	$((0.31221, 0.27503), (0.08936, 0.26719), (0.24528, 0.46698))$
$\$3$	$((0.52264, 0.16079), (0.09843, 0.23925), (0.33266, 0.45820))$
$\$4$	$((0.42599, 0.20126), (0.20440, 0.25183), (0.52297, 0.65032))$

Step 5. In order to determine the score values for each alternative, the scoring definitions were consulted:

- **By using $SFZNOWA_{AA}$:** $\phi(\$1) = 0.65278$, $\phi(\$2) = 0.70068$, $\phi(\$3) = 0.66389$, and $\phi(\$4) = 0.61380$.
- **By using $SFZNWG_{AA}$:** $\phi(\$1) = 0.63543$, $\phi(\$2) = 0.68003$, $\phi(\$3) = 0.64387$, and $\phi(\$4) = 0.60351$.
- **By using $SFZNOWA_{AA}$:** $\phi(\$1) = 0.67008$, $\phi(\$2) = 0.71222$, $\phi(\$3) = 0.66429$, and $\phi(\$4) = 0.62017$.
- **By using $SFZNOWG_{AA}$:** $\phi(\$1) = 0.65675$, $\phi(\$2) = 0.69109$, $\phi(\$3) = 0.64448$, and $\phi(\$4) = 0.61042$.
- **By using $SFZHNWA_{AA}$:** $\phi(\$1) = 0.704218$, $\phi(\$2) = 0.74924$, $\phi(\$3) = 0.70238$, and $\phi(\$4) = 0.66447$.
- **By using $SFZNHWG_{AA}$:** $\phi(\$1) = 0.60073$, $\phi(\$2) = 0.64915$, $\phi(\$3) = 0.63602$, and $\phi(\$4) = 0.56472$.

Step 6. All possibilities are shown in Table 10. They were ranked in descending order.

Table 10
Ranking of the $SFZN_{AA}$ operators

Sr.	$SFZN_{AA}$ operator	Ranking
1	$SFZNOWA_{AA}$	$\phi(\$2) \geq \phi(\$3) \geq \phi(\$1) \geq \phi(\$4)$
2	$SFZNWG_{AA}$	$\phi(\$2) \geq \phi(\$3) \geq \phi(\$1) \geq \phi(\$4)$
3	$SFZNOWA_{AA}$	$\phi(\$2) \geq \phi(\$1) \geq \phi(\$3) \geq \phi(\$4)$
4	$SFZNOWG_{AA}$	$\phi(\$2) \geq \phi(\$1) \geq \phi(\$3) \geq \phi(\$4)$
5	$SFZHNWA_{AA}$	$\phi(\$2) \geq \phi(\$1) \geq \phi(\$3) \geq \phi(\$4)$
6	$SFZNHWG_{AA}$	$\phi(\$2) \geq \phi(\$3) \geq \phi(\$1) \geq \phi(\$4)$

The comparison of the ordering of $SFZN$ and $SFZN_{AA}$ are presented in Table 11.

Figure 3 shows a graphical representation of a comparison between $SFZN$ and $SFZN_{AA}$ ranking.

In the domain of $SFZN$, a thorough study was carried out to assess how well the proposed algorithms performed in comparison to existing measures. The ranking order did slightly shift, but the last best selection remained the same across all approaches used. A detailed comparison of rankings and visualizations employing numerous operators including $SFZNOWA_{AA}$, $SFZNWG_{AA}$, $SFZNOWA_{AA}$, $SFZNOWG_{AA}$, $SFZHNWA_{AA}$, and $SFZNHWG_{AA}$ is given in Figure 3.

In order to assess and contrast the performance of the algorithms proposed with that of pre-existing spherical fuzzy Z number information [24], an exhaustive empirical inquiry was undertaken. Determining the most efficacious decision-making approach in the given context constituted the principal aim of this research. During the course of the investigation, it was noted that the ranking order of alternatives could potentially differ marginally due to the aggregation operators that were implemented. Nevertheless, an exceptional degree of uniformity manifested itself in the determination of the most advantageous course of action among all methodologies. Consequently, $\phi(\$2)$ surfaced

Table 11
Comparison between the $SFZN$ and $SFZN_{AA}$ operators

Sr.	$SFZN$ operators [21]	$\phi(\$_1)$	$\phi(\$_2)$	$\phi(\$_3)$	$\phi(\$_4)$	Ranking
1	$SFZNWA$	0.65984	0.71222	0.66430	0.61687	$\phi(\$_2) \geq \phi(\$_3) \geq \phi(\$_1) \geq \phi(\$_4)$
2	$SFZNGW$	0.64307	0.69109	0.64449	0.60684	$\phi(\$_2) \geq \phi(\$_3) \geq \phi(\$_1) \geq \phi(\$_4)$
3	$SFZNOWA$	0.66734	0.70069	0.66384	0.61875	$\phi(\$_2) \geq \phi(\$_1) \geq \phi(\$_3) \geq \phi(\$_4)$
4	$SFZNOWG$	0.65675	0.69109	0.64448	0.61042	$\phi(\$_2) \geq \phi(\$_1) \geq \phi(\$_3) \geq \phi(\$_4)$
5	$SFZNHWA$	0.64370	0.68348	0.63213	0.58342	$\phi(\$_2) \geq \phi(\$_1) \geq \phi(\$_3) \geq \phi(\$_4)$
6	$SFZNHWG$	0.62785	0.67486	0.64326	0.60092	$\phi(\$_2) \geq \phi(\$_3) \geq \phi(\$_1) \geq \phi(\$_4)$
Sr.	$SFZN_{AA}$ operator	$\phi(\$_1)$	$\phi(\$_2)$	$\phi(\$_3)$	$\phi(\$_4)$	Ranking
1	$SFZNWA_{AA}$	0.65278	0.70068	0.66389	0.61380	$\phi(\$_2) \geq \phi(\$_3) \geq \phi(\$_1) \geq \phi(\$_4)$
2	$SFZNGW_{AA}$	0.63543	0.68003	0.64387	0.60351	$\phi(\$_2) \geq \phi(\$_3) \geq \phi(\$_1) \geq \phi(\$_4)$
3	$SFZNOWA_{AA}$	0.67008	0.71222	0.66429	0.62017	$\phi(\$_2) \geq \phi(\$_1) \geq \phi(\$_3) \geq \phi(\$_4)$
4	$SFZNOWG_{AA}$	0.65491	0.67891	0.64960	0.604377	$\phi(\$_2) \geq \phi(\$_1) \geq \phi(\$_3) \geq \phi(\$_4)$
5	$SFZNHWA_{AA}$	0.704218	0.74924	0.70238	0.66447	$\phi(\$_2) \geq \phi(\$_1) \geq \phi(\$_3) \geq \phi(\$_4)$
6	$SFZNHWG_{AA}$	0.60073	0.64915	0.63602	0.56472	$\phi(\$_2) \geq \phi(\$_3) \geq \phi(\$_1) \geq \phi(\$_4)$

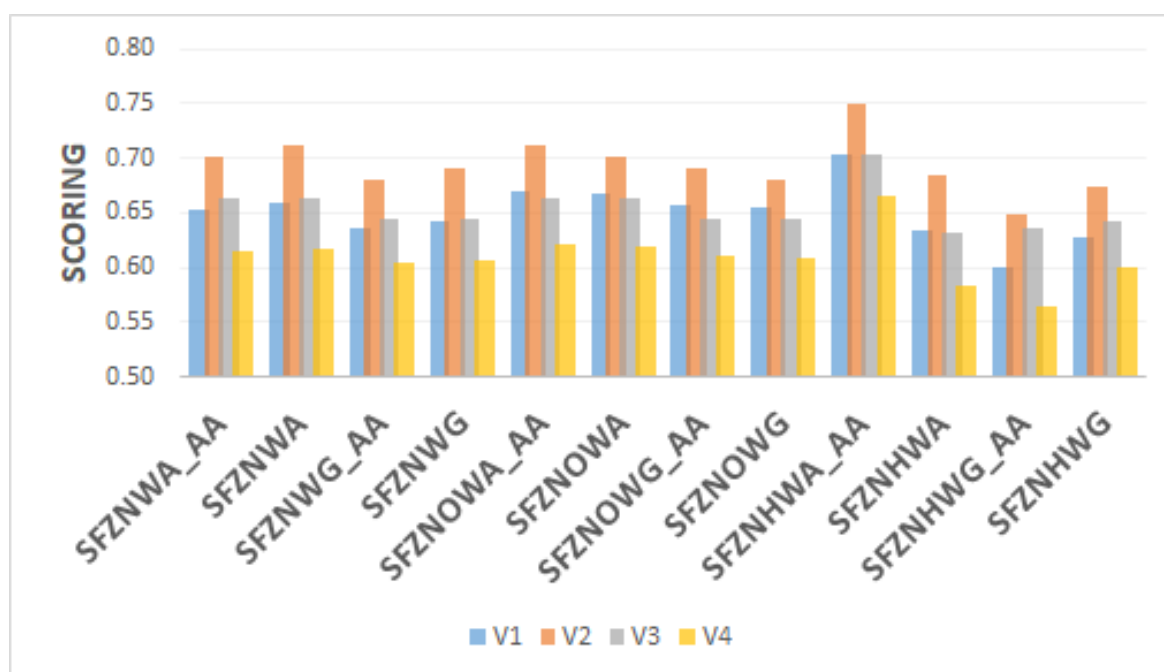


Fig. 3. Graphical representation of the comparison between $SFZN$ and $SFZN_{AA}$ rankings

as the most advantageous alternative course of action. The algorithms that have been proposed are shown to be reliable and strong by the remarkable consistency through which the same best decision is always chosen. Despite some differences in ranks, however small, the same best option being chosen by all shows how good the method is for dealing with problems related to spherical fuzzy Z -number. To further validate the reliability and stability of the proposed methodology, a comparative analysis is conducted using the same decision-making dataset considered in the existing $SFZN$ -based study. In the existing study, the $SFZN$ information was aggregated using the Sugeno-Weber aggregation operator, and the Grey Relational Analysis (GRA) method was subsequently employed to determine the ranking of the alternatives [38]. In the present study, the same decision information is processed using the proposed Aczel–Alsina-based aggregation operator and the developed decision-making framework. Thus, the comparison is performed under the same decision-making environment, while the aggregation mechanism and ranking procedure differ. The final rankings obtained from the two ap-

proaches are presented in Table 12. The results demonstrate that the proposed approach produces a ranking that is consistent with the ranking obtained using the existing Sugeno–Weber and GRA-based approach. In particular, the most desirable alternative identified by the proposed methodology is consistent with the result of the existing approach. Such consistency is important because the two approaches employ different aggregation mechanisms for processing *SFZN* information. The proposed Aczel–Alsina aggregation operator therefore demonstrates its capability to preserve the essential preference relationships among the alternatives while providing an alternative mechanism for aggregating uncertain and imprecise *SFZN* information. Furthermore, Figure 4 graphically illustrates

Table 12

Comparison of the alternative rankings obtained by the existing Sugeno Weber –GRA approach and the proposed Aczel–Alsina approach

Sr.	Sugeno Weber operators [38]	Scoring
1	$SFZNWA_{SW}$	$\phi(\$_1) \geq \phi(\$_2) \geq \phi(\$_4) \geq \phi(\$_3)$
2	$SFZNWG_{SW}$	$\phi(\$_1) \geq \phi(\$_2) \geq \phi(\$_4) \geq \phi(\$_3)$
3	$SFZNOWA_{SW}$	$\phi(\$_1) \geq \phi(\$_2) \geq \phi(\$_4) \geq \phi(\$_3)$
4	$SFZNOWG_{SW}$	$\phi(\$_1) \geq \phi(\$_2) \geq \phi(\$_4) \geq \phi(\$_3)$
5	$SFZNHWA_{SW}$	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
6	$SFZNHWG_{SW}$	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
7	GRA method	$\wp_1 \geq \wp_3 \geq \wp_2 \geq \wp_4$
Sr.	Operators	Scoring
1	$SFZNWA_{AA}$	$\phi(\$_1) \geq \phi(\$_2) \geq \phi(\$_4) \geq \phi(\$_3)$
2	$SFZNWG_{AA}$	$\phi(\$_1) \geq \phi(\$_2) \geq \phi(\$_4) \geq \phi(\$_3)$
3	$SFZNOWA_{AA}$	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
4	$SFZNOWG_{AA}$	$\phi(\$_1) \geq \phi(\$_2) \geq \phi(\$_3) \geq \phi(\$_4)$
5	$SFZNHWA_{AA}$	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
6	$SFZNHWG_{AA}$	$\phi(\$_1) \geq \phi(\$_2) \geq \phi(\$_4) \geq \phi(\$_3)$

the rankings generated by the two approaches. The close correspondence between the two ranking profiles provides additional evidence regarding the stability of the proposed methodology. It should be emphasized that this comparison is intended to examine the consistency and robustness of the proposed framework rather than to claim that either approach is universally superior. The agreement of the final rankings provides supportive evidence that the proposed Aczel–Alsina-based framework can generate reliable decision outcomes for the considered logistics and supply-chain decision-making problem.

7.3 Sensitivity Analysis with Respect to Criterion Weights

To investigate the robustness of the proposed decision-making framework against variations in the criterion weights, a sensitivity analysis was performed. Since the relative importance of the criteria may influence the final ranking of alternatives, different weight configurations were considered while keeping the decision-making information and the proposed aggregation mechanism unchanged. To construct the sensitivity scenarios, the weight of each criterion was systematically increased or decreased by a predetermined percentage, while the remaining weights were adjusted proportionally so that the normalization condition remained satisfied.

In this study, several weight configurations were generated by considering moderate positive and negative variations in the criterion weights. For each scenario, the proposed *SFZN*–Aczel–Alsina aggregation procedure was repeated, and the resulting preference scores and rankings of the alternatives were recorded.

The results of the sensitivity analysis are presented in Table 13. The comparison of the ranking

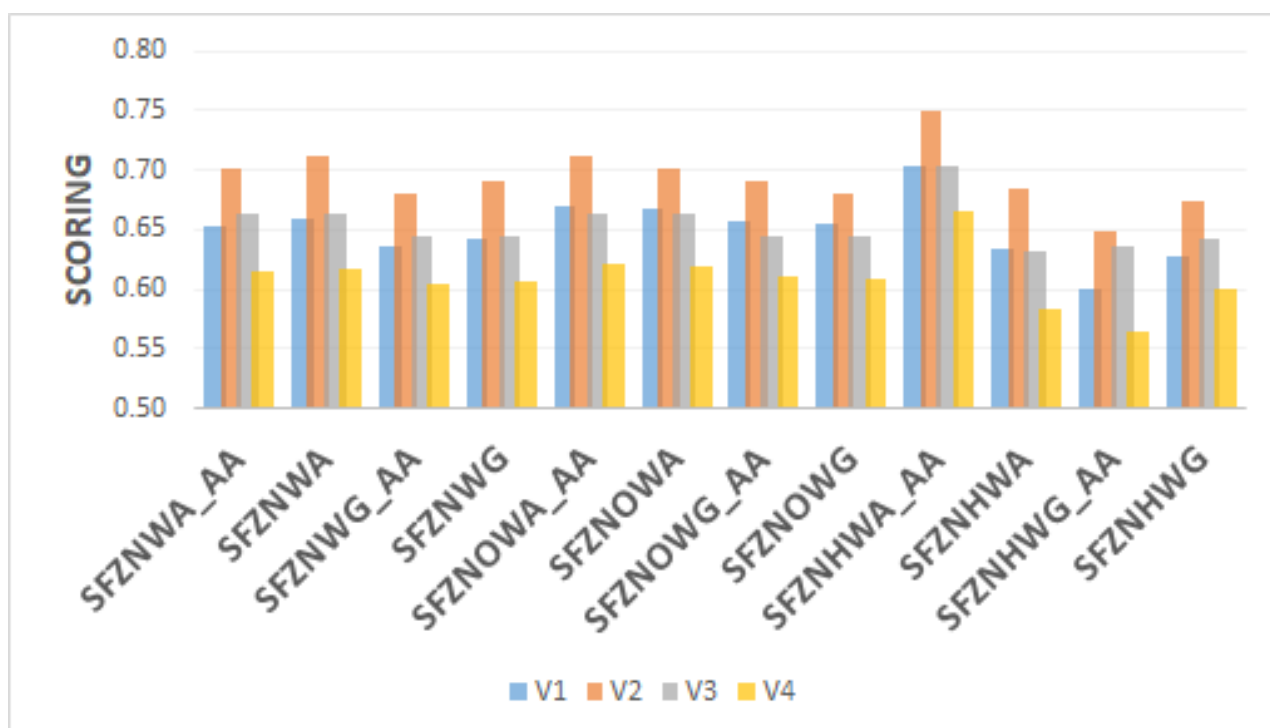


Fig. 4. Comparison of the rankings obtained by the existing Sugeno–Weber–GRA approach and the proposed Aczel–Alsina approach

results under different weight configurations demonstrates the degree to which the final decision is affected by changes in the relative importance of the criteria. If the most preferred alternative remains unchanged across the considered scenarios, the proposed framework can be regarded as robust with respect to moderate variations in criterion weights. Conversely, any changes in the ranking order indicate the criteria that have a greater influence on the final decision.

Overall, the sensitivity analysis provides additional evidence regarding the stability of the proposed methodology and demonstrates whether the final decision is dependent on a particular weight configuration.

8. Conclusion

The core objective of this study was to establish fundamental operational guidelines for $SFZN$ based on the AA operator. Subsequently, leveraging these proposed operational principles, a set of new aggregation operators, namely $SFZNSA_{AA}$, $SFZNSA_{AA}$, $SFZNSHA_{AA}$, $SFZNSG_{AA}$, $SFZNSG_{AA}$, and $SFZNSHG_{AA}$ were formulated. This paper explored new kinds of aggregation operators by providing their important attributes, theorems and peculiarities. MAGDM problems have unique situations that require an effective decision-making technique which makes use of these operators called endorsements. The inclusion of a parameter in the decision-making process has allowed for the consideration of both positive and negative factors while making decisions. The research methods employed in this study included the development of these novel aggregation operators and their application in a decision-making framework for real-time logistics tracking and quality control scenarios. The integration of SFSs and fuzzy Z-number with the $SFZN_{AA}$ operators marks an innovation in this research, which offers a superior and more inclusive decision-making framework when compared to already existing methods.

The proposed $SFZN_{AA}$ operators were confirmed through empirical proof obtained through extensive quality control tracking in real time and through usage of logistics and comparisons with other existing methods. Furthermore, mathematicians further expounded on their findings with rigorous proof.

Nevertheless, there are some shortcomings that make it almost impossible to conduct real-time

Table 13
Sensitivity analysis of the proposed approach under different attribute weights

Weights	Operators	Score Values				Ranking Order
		$\mathbb{R}_1$	$\mathbb{R}_2$	$\mathbb{R}_3$	$\mathbb{R}_4$	
$\Omega_1=0.148$	$SFZNWA_{AA}$	0.66480	0.66238	0.64082	0.65206	$\phi(\$_1) \geq \phi(\$_2) \geq \phi(\$_4) \geq \phi(\$_3)$
$\Omega_2=0.279$	$SFZNWG_{AA}$	0.65168	0.63789	0.63111	0.63395	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
$\Omega_3=0.292$	$SFZNOWA_{AA}$	0.64962	0.64090	0.63816	0.64389	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
$\Omega_4=0.280$	$SFZNOWG_{AA}$	0.64024	0.63410	0.62847	0.62589	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
–	$SFZNHWA_{AA}$	0.64269	0.63263	0.62931	0.63632	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
–	$SFZNHWG_{AA}$	0.64465	0.62960	0.62534	0.63163	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
$\Omega_1=0.168$	$SFZNWA_{AA}$	0.67148	0.66538	0.62082	0.66316	$\phi(\$_1) \geq \phi(\$_2) \geq \phi(\$_4) \geq \phi(\$_3)$
$\Omega_2=0.269$	$SFZNWG_{AA}$	0.65168	0.63789	0.63111	0.63395	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
$\Omega_3=0.282$	$SFZNOWA_{AA}$	0.65962	0.63910	0.63816	0.65389	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
$\Omega_4=0.281$	$SFZNOWG_{AA}$	0.64824	0.63510	0.61845	0.62461	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
–	$SFZNHWA_{AA}$	0.65269	0.62263	0.60931	0.64641	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
–	$SFZNHWG_{AA}$	0.66465	0.65260	0.62534	0.66163	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
$\Omega_1=0.138$	$SFZNWA_{AA}$	0.68348	0.66538	0.62082	0.66316	$\phi(\$_1) \geq \phi(\$_2) \geq \phi(\$_4) \geq \phi(\$_3)$
$\Omega_2=0.289$	$SFZNWG_{AA}$	0.65168	0.63789	0.63111	0.63395	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
$\Omega_3=0.302$	$SFZNOWA_{AA}$	0.65962	0.63910	0.63816	0.65389	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
$\Omega_4=0.271$	$SFZNOWG_{AA}$	0.65724	0.63510	0.61845	0.62779	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
–	$SFZNHWA_{AA}$	0.66269	0.62263	0.60931	0.65641	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$
–	$SFZNHWG_{AA}$	0.66465	0.65260	0.62534	0.66163	$\phi(\$_1) \geq \phi(\$_4) \geq \phi(\$_2) \geq \phi(\$_3)$

tracking in logistics such as uncertainty in data collected or changing patterns of logistical activities, necessitating constant refinement of these proposed techniques. Future studies will focus on coming up with new decision-making frameworks which will be applicable within the context of $SFZN_{AA}$ by including different methods like TOPSIS, EDAS, and ELECTRE among others to improve upon decision-making capabilities. Thus, addressing these limitations and considering new methodologies could contribute towards improving further on this process called decision-making within real time logistic tracking as well as other MAGDM problems.

8.1 Limitations and Future Research Directions

Although the proposed methodology provides an effective framework for handling uncertainty in multi-criteria decision-making, it has some limitations. First, the quality of the obtained results may depend on the reliability and subjectivity of expert evaluations. Second, the proposed fuzzy aggregation procedure involves relatively complex mathematical operations, which may increase the computational burden when the number of alternatives and attributes becomes large. Third, the final ranking may be influenced by the selection and weighting of the decision criteria. Therefore, future studies may focus on developing more computationally efficient formulations, incorporating automated or data-driven weighting mechanisms, and validating the proposed framework through larger-scale real-world applications. Furthermore, the proposed approach can be extended to other fuzzy environments and integrated with additional MCDM techniques to further examine its robustness and applicability.

Appendix

Proof. Proof of Theorem 2.

We will show this by mathematical induction, following Operations (1) and (3) of Definition 10 where $n = 2$:

$$\begin{aligned}
\Omega_1 \odot \mathfrak{S}_{\kappa_1} &= \left\{ \begin{aligned} &\left(\sqrt{1 - e^{-(\Omega_1(-\ln(1-(\sigma_1)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{1 - e^{-(\Omega_1(-\ln(1-(\tau_{\sigma_1})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \\ &\left(\sqrt{e^{-(\Omega_1(-\ln(\mu_1)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{e^{-(\Omega_1(-\ln(\tau_{\mu_1})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \\ &\left(\sqrt{e^{-(\Omega_1(-\ln(\psi_1)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{e^{-(\Omega_1(-\ln(\tau_{\psi_1})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \end{aligned} \right\}. \\
\Omega_2 \odot \mathfrak{S}_{\kappa_2} &= \left\{ \begin{aligned} &\left(\sqrt{1 - e^{-(\Omega_2(-\ln(1-(\sigma_2)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{1 - e^{-(\Omega_2(-\ln(1-(\tau_{\sigma_2})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \\ &\left(\sqrt{e^{-(\Omega_2(-\ln(\mu_2)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{e^{-(\Omega_2(-\ln(\tau_{\mu_2})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \\ &\left(\sqrt{e^{-(\Omega_2(-\ln(\psi_2)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{e^{-(\Omega_2(-\ln(\tau_{\psi_2})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \end{aligned} \right\}. \\
SFZNA_{AA}(\mathfrak{S}_{\kappa_1} \oplus \mathfrak{S}_{\kappa_2}) &= \sum_{\# = 1}^2 \Omega_{\mathfrak{S}_{\kappa_{\#}}} = \Omega_1 \mathfrak{S}_{\kappa_1} \oplus \Omega_2 \mathfrak{S}_{\kappa_2} \\
&= \left\{ \begin{aligned} &\left(\sqrt{1 - e^{-(\Omega_1(-\ln(1-(\sigma_1)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{1 - e^{-(\Omega_1(-\ln(1-(\tau_{\sigma_1})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \\ &\left(\sqrt{e^{-(\Omega_1(-\ln(\mu_1)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{e^{-(\Omega_1(-\ln(\tau_{\mu_1})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \\ &\left(\sqrt{e^{-(\Omega_1(-\ln(\psi_1)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{e^{-(\Omega_1(-\ln(\tau_{\psi_1})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \end{aligned} \right\} \oplus \\
&\left\{ \begin{aligned} &\left(\sqrt{1 - e^{-(\Omega_2(-\ln(1-(\sigma_2)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{1 - e^{-(\Omega_2(-\ln(1-(\tau_{\sigma_2})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \\ &\left(\sqrt{e^{-(\Omega_2(-\ln(\mu_2)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{e^{-(\Omega_2(-\ln(\tau_{\mu_2})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \\ &\left(\sqrt{e^{-(\Omega_2(-\ln(\psi_2)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{e^{-(\Omega_2(-\ln(\tau_{\psi_2})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \end{aligned} \right\}. \\
\mathfrak{S}_{\kappa_1} \oplus \mathfrak{S}_{\kappa_2} &= \left\{ \begin{aligned} &\left(\sqrt{1 - e^{-(\Omega_1(-\ln(1-(\sigma_1)^2))^{\vartheta} + \Omega_2(-\ln(1-(\sigma_2)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{1 - e^{-(\Omega_1(-\ln(1-(\tau_{\sigma_1})^2))^{\vartheta} + \Omega_2(-\ln(1-(\tau_{\sigma_2})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \\ &\left(\sqrt{e^{-(\Omega_1(-\ln(\mu_1)^2))^{\vartheta} + \Omega_2(-\ln(\mu_2)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{e^{-(\Omega_1(-\ln(\tau_{\mu_1})^2))^{\vartheta} + \Omega_2(-\ln(\tau_{\mu_2})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \\ &\left(\sqrt{e^{-(\Omega_1(-\ln(\psi_1)^2))^{\vartheta} + \Omega_2(-\ln(\psi_2)^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{e^{-(\Omega_1(-\ln(\tau_{\psi_1})^2))^{\vartheta} + \Omega_2(-\ln(\tau_{\psi_2})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \end{aligned} \right\}. \\
&= \left\{ \begin{aligned} &\left(\sqrt{1 - e^{-(\sum_{\# = 1}^2 \Omega_{\#}(-\ln(1-(\sigma_{\#})^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{1 - e^{-(\sum_{\# = 1}^2 \Omega_{\#}(-\ln(1-(\tau_{\sigma_{\#}})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \\ &\left(\sqrt{e^{-(\sum_{\# = 1}^2 \Omega_{\#}(-\ln(\mu_{\#})^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{e^{-(\sum_{\# = 1}^2 \Omega_{\#}(-\ln(\tau_{\mu_{\#}})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \\ &\left(\sqrt{e^{-(\sum_{\# = 1}^2 \Omega_{\#}(-\ln(\psi_{\#})^2))^{\vartheta})^{\frac{1}{\vartheta}}}}, \sqrt{e^{-(\sum_{\# = 1}^2 \Omega_{\#}(-\ln(\tau_{\psi_{\#}})^2))^{\vartheta})^{\frac{1}{\vartheta}}}} \right) \end{aligned} \right\}.
\end{aligned}$$

A reasoning was further provided for $n=2$ in this regard. Moreover, we also take into account that it applies to $n = \beta$.

$$SFZNA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_{\beta}}) = \sum_{\# = 1}^{\beta} \Omega_i \mathfrak{S}_{\kappa_{\#}}$$

$$= \left\{ \begin{array}{l} \left(\sqrt{1 - e^{-(\sum_{\# = 1}^{\beta} \Omega_{\#}(-\ln(1-(\sigma_{\#}^2))^{\varphi})^{\frac{1}{\varphi}})}, \sqrt{1 - e^{-(\sum_{\# = 1}^{\beta} \Omega_{\#}(-\ln(1-(\tau_{\sigma_{\#}}^2))^{\varphi})^{\frac{1}{\varphi}})} \right) \\ \left(\sqrt{e^{-(\sum_{\# = 1}^{\beta} \Omega_{\#}(-\ln(\mu_{\#}^2))^{\varphi})^{\frac{1}{\varphi}}}, \sqrt{e^{-(\sum_{\# = 1}^{\beta} \Omega_{\#}(-\ln(\tau_{\mu_{\#}}^2))^{\varphi})^{\frac{1}{\varphi}})} \right) \\ \left(\sqrt{e^{-(\sum_{\# = 1}^{\beta} \Omega_{\#}(-\ln(\psi_{\#}^2))^{\varphi})^{\frac{1}{\varphi}}}, \sqrt{e^{-(\sum_{\# = 1}^{\beta} \Omega_{\#}(-\ln(\tau_{\psi_{\#}}^2))^{\varphi})^{\frac{1}{\varphi}})} \right) \end{array} \right\}.$$

After this, we will demonstrate this for $n = \beta + 1$ and we have that:

$$SFZNSWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_{\beta+1}}) = \sum_{\# = 1}^{\beta} \Omega_{\#} \mathfrak{S}_{\kappa_{\#}} \oplus \Omega_{\beta+1} \mathfrak{S}_{\kappa_{\beta+1}}$$

$$= \left\{ \begin{array}{l} \left(\sqrt{1 - e^{-(\sum_{\# = 1}^{\beta+1} \Omega_{\#}(-\ln(1-(\sigma_{\#}^2))^{\varphi})^{\frac{1}{\varphi}})}, \sqrt{1 - e^{-(\sum_{\# = 1}^{\beta+1} \Omega_{\#}(-\ln(1-(\tau_{\sigma_{\#}}^2))^{\varphi})^{\frac{1}{\varphi}})} \right) \\ \left(\sqrt{e^{-(\sum_{\# = 1}^{\beta+1} \Omega_{\#}(-\ln(\mu_{\#}^2))^{\varphi})^{\frac{1}{\varphi}}}, \sqrt{e^{-(\sum_{\# = 1}^{\beta+1} \Omega_{\#}(-\ln(\tau_{\mu_{\#}}^2))^{\varphi})^{\frac{1}{\varphi}})} \right) \\ \left(\sqrt{e^{-(\sum_{\# = 1}^{\beta+1} \Omega_{\#}(-\ln(\psi_{\#}^2))^{\varphi})^{\frac{1}{\varphi}}}, \sqrt{e^{-(\sum_{\# = 1}^{\beta+1} \Omega_{\#}(-\ln(\tau_{\psi_{\#}}^2))^{\varphi})^{\frac{1}{\varphi}})} \right) \end{array} \right\}.$$

This suggests that $\beta + 1$ is valid. Therefore, it is so for all n , thus concluding the argument. $\square$

Proof. Proof of Property 1.

In case $\mathfrak{S}_{\kappa_i}$ are the same, we have:

$$SFZNSWA_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) = \sum_{\# = 1}^n \omega_{\#} \mathfrak{S}_{\kappa}$$

$$= \left\{ \begin{array}{l} \left(\sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(1-(\sigma_{\#}^2))^{\varphi})^{\frac{1}{\varphi}})}, \sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(1-(\tau_{\sigma_{\#}}^2))^{\varphi})^{\frac{1}{\varphi}})} \right) \\ \left(\sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(\mu_{\#}^2))^{\varphi})^{\frac{1}{\varphi}}}, \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(\tau_{\mu_{\#}}^2))^{\varphi})^{\frac{1}{\varphi}})} \right) \\ \left(\sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(\psi_{\#}^2))^{\varphi})^{\frac{1}{\varphi}}}, \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(\tau_{\psi_{\#}}^2))^{\varphi})^{\frac{1}{\varphi}})} \right) \end{array} \right\}.$$

$$= \left\{ \begin{array}{l} \left(\sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(1-(\sigma_{\#}^2))^{\varphi})^{\frac{1}{\varphi}})}, \sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(1-(\tau_{\sigma_{\#}}^2))^{\varphi})^{\frac{1}{\varphi}})} \right) \\ \left(\sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(\mu_{\#}^2))^{\varphi})^{\frac{1}{\varphi}}}, \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(\tau_{\mu_{\#}}^2))^{\varphi})^{\frac{1}{\varphi}})} \right) \\ \left(\sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(\psi_{\#}^2))^{\varphi})^{\frac{1}{\varphi}}}, \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(\tau_{\psi_{\#}}^2))^{\varphi})^{\frac{1}{\varphi}})} \right) \end{array} \right\} = \mathfrak{S}_{\kappa}$$

$\square$

Proof. Proof of Property 2.

As $\mathfrak{S}_{\kappa_i} \subseteq \mathfrak{S}_{\kappa'_i}$, thus $(\sigma_{\#}, \tau_{\sigma_{\#}}) \leq (\sigma_{\#'}, \tau_{\sigma_{\#'}})$, $(\mu_{\#}, \tau_{\mu_{\#}}) \leq (\mu_{\#'}, \tau_{\mu_{\#'}})$, and $(\psi_{\#}, \tau_{\psi_{\#}}) \geq (\psi_{\#'}, \tau_{\psi_{\#'}})$. This implies that:

$$\sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(1-(\sigma_{\#}^2))^{\varphi})^{\frac{1}{\varphi}})} \leq \sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#'}(-\ln(1-(\sigma_{\#'}^2))^{\varphi})^{\frac{1}{\varphi}})},$$

$$\sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(1-(\tau_{\sigma_{\#}}^2))^{\varphi})^{\frac{1}{\varphi}})} \leq \sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#'}(-\ln(1-(\tau_{\sigma_{\#'}}^2))^{\varphi})^{\frac{1}{\varphi}})},$$

$$\sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#}(-\ln(\mu_{\#}^2))^{\varphi})^{\frac{1}{\varphi}})} \leq \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#'}(-\ln(\mu_{\#'}^2))^{\varphi})^{\frac{1}{\varphi}}},$$

$$\begin{aligned}\sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\tau_{\mu_{\#}})^2)^{\varphi})^{\frac{1}{\varphi}}}} &\leq \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#'} (-\ln(\tau_{\mu_{\#'}})^2)^{\varphi})^{\frac{1}{\varphi}}}}, \\ \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\psi_{\#})^2)^{\varphi})^{\frac{1}{\varphi}}}} &\geq \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#'} (-\ln(\psi_{\#'})^2)^{\varphi})^{\frac{1}{\varphi}}}}, \\ \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\tau_{\psi_{\#}})^2)^{\varphi})^{\frac{1}{\varphi}}}} &\geq \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#'} (-\ln(\tau_{\psi_{\#'}})^2)^{\varphi})^{\frac{1}{\varphi}}}},\end{aligned}$$

Besides:

$$\left\{ \begin{aligned} &\left(\sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(1 - (\sigma_{\#}^2)^2)^{\varphi})^{\frac{1}{\varphi}}}}}, \sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(1 - (\tau_{\sigma_{\#}}^2)^2)^{\varphi})^{\frac{1}{\varphi}}}} \right) \\ &\left(\sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\mu_{\#})^2)^{\varphi})^{\frac{1}{\varphi}}}}, \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\tau_{\mu_{\#}})^2)^{\varphi})^{\frac{1}{\varphi}}}} \right) \\ &\left(\sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\psi_{\#})^2)^{\varphi})^{\frac{1}{\varphi}}}}, \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#} (-\ln(\tau_{\psi_{\#}})^2)^{\varphi})^{\frac{1}{\varphi}}}} \right) \end{aligned} \right\} \leq$$

$$\left\{ \begin{aligned} &\left(\sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#'} (-\ln(1 - (\sigma_{\#'}^2)^2)^{\varphi})^{\frac{1}{\varphi}}}}}, \sqrt{1 - e^{-(\sum_{\# = 1}^n \Omega_{\#'} (-\ln(1 - (\tau_{\sigma_{\#'}}^2)^2)^{\varphi})^{\frac{1}{\varphi}}}} \right) \\ &\left(\sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#'} (-\ln(\mu_{\#'})^2)^{\varphi})^{\frac{1}{\varphi}}}}, \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#'} (-\ln(\tau_{\mu_{\#'}})^2)^{\varphi})^{\frac{1}{\varphi}}}} \right) \\ &\left(\sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#'} (-\ln(\psi_{\#'})^2)^{\varphi})^{\frac{1}{\varphi}}}}, \sqrt{e^{-(\sum_{\# = 1}^n \Omega_{\#'} (-\ln(\tau_{\psi_{\#'}})^2)^{\varphi})^{\frac{1}{\varphi}}}} \right) \end{aligned} \right\}$$

Hence:

$$SFZNW A_{AA}(\mathfrak{S}_{\kappa_1}, \mathfrak{S}_{\kappa_2}, \mathfrak{S}_{\kappa_3}, \dots, \mathfrak{S}_{\kappa_n}) \leq SFZNW A_{Ap}(\mathfrak{S}_{\kappa'_1}, \mathfrak{S}_{\kappa'_2}, \mathfrak{S}_{\kappa'_3}, \dots, \mathfrak{S}_{\kappa'_n})$$

□

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Conflicts of Interest

The authors declare no conflicts of interest.

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