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A Distance Measure of Fermatean Fuzzy Sets Based on Triangular Divergence and Its Application in Medical Diagnosis

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ABSTRACT

Fermatean fuzzy sets (FFSs), as one of the representative variants of fuzzy sets, have broad application prospects. FFSs have advantages in modeling uncertain information and therefore have been widely applied. However, how to perfectly quantify the differences between FFS remains an open question. This paper introduces a new distance measure for FFSs, utilizing triangular divergence. The proposed measure is designed to rectify the limitations in the current measure, offering a more effective solution for analyzing FFSs. Moreover, we demonstrate that the proposed distance measure satisfies some basic properties and further show its effectiveness through several numerical examples. Finally, we explore the performance of the proposed distance measure in a medical diagnosis application, and the results show that the proposed distance measure can well overcome the limitations of the current measure.

1. Introduction

With the rapid development of information technology, the uncertainty and ambiguity of data have become more and more obvious, which has prompted the emergence of many theories to deal with imperfections [1,2,3,4,5]. Fuzzy sets, originally proposed by Zadeh [6], provide an effective mathematical framework for processing uncertainty information. Since then, fuzzy sets have gained much attention from researchers and applied to different fields [7,8,9]. After fuzzy sets, Atanassov [10] introduced the notion of intuitionistic fuzzy sets (IFSs), which encompass both the degrees of membership and non-membership, offering a richer portrayal of uncertainty. IFSs have been applied widely, including in fields such as decision making [11,12], pattern recognition [13,14,15,16] and medical diagnosis [17,18,19].

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Subsequently, Yager [20] expanded on IFSs with the development of Pythagorean fuzzy sets (PFSs), where the sum of the squares of degrees of membership and non-membership is constrained to not exceed 1, allowing for a nuanced representation of imperfect information. This led to a surge in research focusing on the properties and applications of PFSs [21,22,23,24]. On this continuum of enhancing fuzzy sets models, Fermatean fuzzy sets (FFSs) [25] have been introduced as a further extension to capture uncertain information with greater precision. FFSs have since ignited considerable interest in the research community [26,27,28], with significant contributions such as Senapati and Yager's development of the score and accurate functions for FFSs, enhancing the comparison of Fermatean fuzzy numbers. Moreover, their creation of various Fermatean fuzzy aggregation operators has been instrumental in addressing multiple-attribute decision-making (MADM) problems [26].

The investigation of distance and similarity measures within the scope of fuzzy sets is a key area that has drawn extensive attention from the academic community. A variety of such measures have been developed specifically for IFSs and PFSs. Szmidt and Kacprzyk [29], for instance, pioneered the use of Hamming and Euclidean distances for calculating the difference between IFSs, while Grzegorzewski [30] applied Hausdorff distance for a similar purpose. Furthering this work, Wang and Xin [31] defined two hybrid distance measures for IFSs and showcased their effectiveness in pattern recognition applications. Song and Wang [32] formulated a similarity measure for IFSs, emphasizing the positive definiteness of the similarity matrix. Zhang and Xu [33] introduced a distance measure for PFSs. However, Ejegwa [34] critiqued this measure for not adhering strictly to the distance axiom and subsequently proposed a normalized version to rectify this. Hussian and Yang [35] further expanded on this concept by adapting both Hamming and Euclidean distances, along with their normalized variants, to differentiate PFSs. Ejegwa and Awolola [36] innovated further by modifying existing distance measures for PFSs, while Xiao and Ding [37] proposed a novel Jensen-Shannon divergence-based measure. Recently, Kumar et al. [38] have developed a new divergence measure for PFSs, aiming to overcome limitations identified in Xiao and Ding's method. Wei and Wei [39] have introduced several similarity measures of PFSs based on the cosine function, and Wang et al. [40] have proposed Dice similarity measures and their generalized versions for PFSs.

To effectively utilize FFSs in practical applications, several distance and similarity measures have been put forward for FFSs. For instance, Kirişci [41] proposed the cosine similarity and distance measures for FFSs and applied them to TOPSIS. Later, Liu and Huang [42] pointed out the limitations of Kirişci's work and presented a new cosine similarity measure. Sahoo [43] presented several similarity measures for FFSs and applied them to group decision-making. Particularly, Deng and Wang [44] proposed a distance measure for FFSs based on triangular divergence. However, Deng and Wang's work exists a serious flaw: in some cases the denominator may be 0, which will render the distance measure invalid. The motivation of this paper is to propose an efficient distance measure to overcome this shortcoming.

In this paper, we propose a new distance measure for FFSs based on the triangular divergence, which can effectively solve the above limitation. The main contributions of this work are:

- A new distance measure for FFSs are proposed and its properties are proved.
- The effectiveness of the proposed distance measure is verified through several numerical examples.
- The proposed distance measure is applied in medical diagnosis to demonstrate its performance.

The rest of this paper is set out as follows. Section 2 reviews the basic concepts of IFSs, PFSs, FFSs and triangular divergence. Section 3 shows the proposed distance measure and demonstrates its properties. In Section 4, a medical diagnosis application is employed to verify the performance of the proposed distance measure. Finally, we make a conclusion in Section 5.

2. Preliminaries

In this section, some basic concepts related to IFSs, PFSs, FFSs and triangular divergence will be given.

2.1 Intuitionistic fuzzy sets

Definition 1 *[10] Let $\mathbb{Z}$ be a finite universe of discourse (UOD). The intuitionistic fuzzy set $\mathcal{A}$ in $\mathbb{Z}$ is defined as:*

$$\mathcal{A} = \{\langle z, \phi_{\mathcal{A}}(z), \varphi_{\mathcal{A}}(z)\rangle | z \in \mathbb{Z}\} \tag{1}$$

where $\phi_{\mathcal{A}}$ and $\varphi_{\mathcal{A}}$ denote the membership and nonmembership degrees of $z \in \mathbb{Z}$. For any $z \in \mathbb{Z}$, $\phi_{\mathcal{A}}(z)$ and $\varphi_{\mathcal{A}}(z)$ meet the following conditions:

$$0 \leq \xi_{\mathcal{A}}(z) + \zeta_{\mathcal{A}}(z) \leq 1 \tag{2}$$

For any $z \in \mathbb{Z}$, the hesitancy degree of the element z is:

$$\psi_{\mathcal{A}}(z) = 1 - \phi_{\mathcal{A}}(z) - \varphi_{\mathcal{A}}(z) \tag{3}$$

2.2 Pythagorean fuzzy sets

Definition 2 *[20] Let $\mathbb{Z}$ be a finite UOD. A Pythagorean fuzzy set $\mathcal{A}$ in $\mathbb{Z}$ is defined as:*

$$\mathcal{A} = \{\langle z, \phi_{\mathcal{A}}(z), \varphi_{\mathcal{A}}(z)\rangle | z \in \mathbb{Z}\} \tag{4}$$

where $\phi_{\mathcal{A}}$ and $\varphi_{\mathcal{A}}$ denote the membership and nonmembership degrees of $z \in \mathbb{Z}$. For any $z \in \mathbb{Z}$, $\phi_{\mathcal{A}}(z)$ and $\varphi_{\mathcal{A}}(z)$ meet the following conditions:

$$0 \leq \phi^2_{\mathcal{A}}(z) + \varphi^2_{\mathcal{A}}(z) \leq 1 \tag{5}$$

For any $z \in \mathbb{Z}$, the hesitancy degree of the element z is:

$$\psi_{\mathcal{A}}(z) = \sqrt{1 - \phi^2_{\mathcal{A}}(z) - \varphi^2_{\mathcal{A}}(z)} \tag{6}$$

2.3 Fermatean fuzzy sets

Definition 3 *[25] Let $\mathbb{Z}$ be a finite UOD. A Fermatean fuzzy set $\mathcal{A}$ in $\mathbb{Z}$ is defined as:*

$$\mathcal{A} = \{\langle z, \phi_{\mathcal{A}}(z), \varphi_{\mathcal{A}}(z)\rangle | z \in \mathbb{Z}\} \tag{7}$$

where $\phi_{\mathcal{A}}$ and $\varphi_{\mathcal{A}}$ denote the membership and nonmembership degrees of $z \in \mathbb{Z}$. For any $z \in \mathbb{Z}$, $\phi_{\mathcal{A}}(z)$ and $\varphi_{\mathcal{A}}(z)$ meet the following conditions:

$$0 \leq \phi^3_{\mathcal{A}}(z) + \varphi^3_{\mathcal{A}}(z) \leq 1 \tag{8}$$

For any $z \in \mathbb{Z}$, the hesitancy degree of the element z is:

$$\psi_{\mathcal{A}}(z) = \sqrt[3]{1 - \phi^3_{\mathcal{A}}(z) - \varphi^3_{\mathcal{A}}(z)} \tag{9}$$

2.4 Triangular divergence-based distance measure of FFSs

Definition 4 *[44] The triangular divergence-based distance measure between $\mathcal{A}$ and $\mathcal{B}$ is defined as:*

$$D_T(\mathcal{A},\mathcal{B}) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left(\frac{\left(\phi_{\mathcal{A}}^3(z_i)-\phi_{\mathcal{B}}^3(z_i)\right)^2}{\phi_{\mathcal{A}}^3(z_i)+\phi_{\mathcal{B}}^3(z_i)}+\frac{\left(\varphi_{\mathcal{A}}^3(z_i))-\varphi_{\mathcal{B}}^3(z_i))\right)^2}{\varphi_{\mathcal{A}}^3(z_i))+\varphi_{\mathcal{B}}^3(z_i))}\right)} \tag{10}$$

Remark 1 *Although the distance measure D_T of FFSs satisfies the axiomatic definitions of distance and can achieve satisfactory results in some cases, D_T still has a serious flaw: when $\phi_{\mathcal{A}}^3(z_i)+\phi_{\mathcal{B}}^3(z_i)=0$ or $\varphi_{\mathcal{A}}^3(z_i)+\varphi_{\mathcal{B}}^3(z_i)=0$, D_T will be invalid.*

3. The proposed distance measure

In this section, we propose a new distance measures for FFSs based on triangular divergence and analyze some interesting properties of the distance measure.

Definition 5 *Let $\mathbb{Z}$ be an UOD. For two FFSs $\mathcal{A}=\{\langle z_i,\phi_{\mathcal{A}}(z_i),\varphi_{\mathcal{A}}(z_i)\rangle | z_i\in\mathbb{Z}\}$ and $\mathcal{B}=\{\langle z_i,\phi_{\mathcal{B}}(z_i),\varphi_{\mathcal{B}}(z_i)\rangle | z_i\in\mathbb{Z}\}$. Then, a distance measure, named $D_{TD}(\mathcal{A},\mathcal{B})$, between FFSs $\mathcal{A}$ and $\mathcal{B}$ is defined as follows:*

$$D_{TD}(\mathcal{A},\mathcal{B}) = \sqrt{\frac{3}{2n}\sum_{i=1}^{n}\left(\frac{\left(\phi_{\mathcal{A}}^3(z_i)-\phi_{\mathcal{B}}^3(z_i)\right)^2}{\phi_{\mathcal{A}}^3(z_i)+\phi_{\mathcal{B}}^3(z_i)+2}+\frac{\left(\varphi_{\mathcal{A}}^3(z_i)-\varphi_{\mathcal{B}}^3(z_i)\right)^2}{\varphi_{\mathcal{A}}^3(z_i)+\varphi_{\mathcal{B}}^3(z_i)+2}\right)} \tag{11}$$

Theorem 1 *Let $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ be three FFSs in UOD $\mathbb{Z}$, D_{TD} has the following properties:*

1. *(Bounded) $0\le D_{TD}(\mathcal{A},\mathcal{B})\le 1$.*
2. *(Separability) $D_{TD}(\mathcal{A},\mathcal{B})=0$ if and only if $\mathcal{A}=\mathcal{B}$.*
3. *(Symmetry) $D_{TD}(\mathcal{A},\mathcal{B})=D_{TD}(\mathcal{B},\mathcal{A})$.*
4. *(Containment) If $\mathcal{A}\subseteq\mathcal{B}\subseteq\mathcal{C}$, then $D_{TD}(\mathcal{A},\mathcal{B})\le D_{TD}(\mathcal{A},\mathcal{C})$ and $D_{TD}(\mathcal{B},\mathcal{C})\le D_{TD}(\mathcal{A},\mathcal{C})$.*

Proof 1 *Given two FFSs $\mathcal{A}$ and $\mathcal{B}$ on UOD $\mathbb{Z}$, we have:*

$$D_{TD}(\mathcal{A},\mathcal{B}) = \sqrt{\frac{3}{2n}\sum_{i=1}^{n}\left(\frac{\left(\phi_{\mathcal{A}}^3(z_i)-\phi_{\mathcal{B}}^3(z_i)\right)^2}{\phi_{\mathcal{A}}^3(z_i)+\phi_{\mathcal{B}}^3(z_i)+2}+\frac{\left(\varphi_{\mathcal{A}}^3(z_i)-\varphi_{\mathcal{B}}^3(z_i)\right)^2}{\varphi_{\mathcal{A}}^3(z_i)+\varphi_{\mathcal{B}}^3(z_i)+2}\right)}$$

We can easily get $D_{TD}(\mathcal{A},\mathcal{B})\ge 0$. Moreover, $0\le\phi_{\mathcal{A}}(z_i),\phi_{\mathcal{B}}(z_i)\le 1$ and $0\le\varphi_{\mathcal{A}}(z_i),\varphi_{\mathcal{B}}(z_i)\le 1$, we thus infer:

$$\begin{aligned}
&0\le\frac{\left(\phi_{\mathcal{A}}^3(z_i)-\phi_{\mathcal{B}}^3(z_i)\right)^2}{\phi_{\mathcal{A}}^3(z_i)+\phi_{\mathcal{B}}^3(z_i)+2}\le\frac{1}{3}\text{ and }0\le\frac{\left(\varphi_{\mathcal{A}}^3(z_i)-\varphi_{\mathcal{B}}^3(z_i)\right)^2}{\varphi_{\mathcal{A}}^3(z_i)+\varphi_{\mathcal{B}}^3(z_i)+2}\le\frac{1}{3}\\
&\Rightarrow 0\le\frac{3}{2n}\sum_{i=1}^{n}\left(\frac{\left(\phi_{\mathcal{A}}^3(z_i)-\phi_{\mathcal{B}}^3(z_i)\right)^2}{\phi_{\mathcal{A}}^3(z_i)+\phi_{\mathcal{B}}^3(z_i)+2}+\frac{\left(\varphi_{\mathcal{A}}^3(z_i)-\varphi_{\mathcal{B}}^3(z_i)\right)^2}{\varphi_{\mathcal{A}}^3(z_i)+\varphi_{\mathcal{B}}^3(z_i)+2}\right)\le 1\\
&\Rightarrow 0\le\sqrt{\frac{3}{2n}\sum_{i=1}^{n}\left(\frac{\left(\phi_{\mathcal{A}}^3(z_i)-\phi_{\mathcal{B}}^3(z_i)\right)^2}{\phi_{\mathcal{A}}^3(z_i)+\phi_{\mathcal{B}}^3(z_i)+2}+\frac{\left(\varphi_{\mathcal{A}}^3(z_i)-\varphi_{\mathcal{B}}^3(z_i)\right)^2}{\varphi_{\mathcal{A}}^3(z_i)+\varphi_{\mathcal{B}}^3(z_i)+2}\right)}\le 1
\end{aligned}$$

Hence, we can obtain $0\le D_{TD}(\mathcal{A},\mathcal{B})\le 1$.

Proof 2 *Given two same FFSs $\mathcal{A}$ and $\mathcal{B}$ on UOD $\mathbb{Z}$, we have $\phi^3_{\mathcal{A}}(z_i) = \phi^3_{\mathcal{B}}(z_i)$ and $\varphi^3_{\mathcal{A}}(z_i) = \varphi^3_{\mathcal{B}}(z_i)$. Hence, we can get:*

$$D_{TD}(\mathcal{A},\mathcal{B}) = \sqrt{\frac{3}{2n}\sum_{i=1}^{n}\left(\frac{\left(\phi^3_{\mathcal{A}}(z_i)-\phi^3_{\mathcal{B}}(z_i)\right)^2}{\phi^3_{\mathcal{A}}(z_i)+\phi^3_{\mathcal{B}}(z_i)+2}+\frac{\left(\varphi^3_{\mathcal{A}}(z_i)-\varphi^3_{\mathcal{B}}(z_i)\right)^2}{\varphi^3_{\mathcal{A}}(z_i)+\varphi^3_{\mathcal{B}}(z_i)+2}\right)}$$
$$\sqrt{\frac{3}{2n}\sum_{i=1}^{n}\left(\frac{\left(\phi^3_{\mathcal{A}}(z_i)-\phi^3_{\mathcal{A}}(z_i)\right)^2}{\phi^3_{\mathcal{A}}(z_i)+\phi^3_{\mathcal{A}}(z_i)+2}+\frac{\left(\varphi^3_{\mathcal{A}}(z_i)-\varphi^3_{\mathcal{A}}(z_i)\right)^2}{\varphi^3_{\mathcal{A}}(z_i)+\varphi^3_{\mathcal{A}}(z_i)+2}\right)} = 0$$

For any $z_i \in \mathbb{Z}$, if $D_{TD}(\mathcal{A},\mathcal{B}) = 0$, we have:

$$D_{TD}(\mathcal{A},\mathcal{B}) = \sqrt{\frac{3}{2n}\sum_{i=1}^{n}\left(\frac{\left(\phi^3_{\mathcal{A}}(z_i)-\phi^3_{\mathcal{B}}(z_i)\right)^2}{\phi^3_{\mathcal{A}}(z_i)+\phi^3_{\mathcal{B}}(z_i)+2}+\frac{\left(\varphi^3_{\mathcal{A}}(z_i)-\varphi^3_{\mathcal{B}}(z_i)\right)^2}{\varphi^3_{\mathcal{A}}(z_i)+\varphi^3_{\mathcal{B}}(z_i)+2}\right)} = 0$$

Thus, we can get:

$$\begin{aligned}
&\frac{\left(\phi^3_{\mathcal{A}}(z_i)-\phi^3_{\mathcal{B}}(z_i)\right)^2}{\phi^3_{\mathcal{A}}(z_i)+\phi^3_{\mathcal{B}}(z_i)+2} = 0 \text{ and } \frac{\left(\varphi^3_{\mathcal{A}}(\mathrm{z_i})-\varphi^3_{\mathcal{B}}(\mathrm{z_i})\right)^2}{\varphi^3_{\mathcal{A}}(\mathrm{z_i})+\varphi^3_{\mathcal{B}}(\mathrm{z_i})+2} = 0\\
&\Rightarrow \left(\phi^3_{\mathcal{A}}(z_i)-\phi^3_{\mathcal{B}}(z_i)\right)^2 = 0 \text{ and } \left(\varphi^3_{\mathcal{A}}(\mathrm{z_i})-\varphi^3_{\mathcal{B}}(\mathrm{z_i})\right)^2 = 0\\
&\Rightarrow \phi^3_{\mathcal{A}}(z_i)-\phi^3_{\mathcal{B}}(z_i) = 0 \text{ and } \varphi^3_{\mathcal{A}}(\mathrm{z_i})-\varphi^3_{\mathcal{B}}(\mathrm{z_i}) = 0\\
&\Rightarrow \phi^3_{\mathcal{A}}(z_i) = \phi^3_{\mathcal{B}}(z_i) \text{ and } \varphi^3_{\mathcal{A}}(\mathrm{z_i}) = \varphi^3_{\mathcal{B}}(\mathrm{z_i})
\end{aligned}$$

Thereby, we can obtain $D_{TD}(\mathcal{A},\mathcal{B}) = 0$ if and only if $\mathcal{A} = \mathcal{B}$.

Proof 3 *Given two FFSs $\mathcal{A}$ and $\mathcal{B}$ on UOD $\mathbb{Z}$, we have:*

$$\begin{aligned}
D_{TD}(\mathcal{A},\mathcal{B}) &= \sqrt{\frac{3}{2n}\sum_{i=1}^{n}\left(\frac{\left(\phi^3_{\mathcal{A}}(z_i)-\phi^3_{\mathcal{B}}(z_i)\right)^2}{\phi^3_{\mathcal{A}}(z_i)+\phi^3_{\mathcal{B}}(z_i)+2}+\frac{\left(\varphi^3_{\mathcal{A}}(z_i)-\varphi^3_{\mathcal{B}}(z_i)\right)^2}{\varphi^3_{\mathcal{A}}(z_i)+\varphi^3_{\mathcal{B}}(z_i)+2}\right)}\\
&= \sqrt{\frac{3}{2n}\sum_{i=1}^{n}\left(\frac{\left(\phi^3_{\mathcal{B}}(z_i)-\phi^3_{\mathcal{A}}(z_i)\right)^2}{\phi^3_{\mathcal{B}}(z_i)+\phi^3_{\mathcal{A}}(z_i)+2}+\frac{\left(\varphi^3_{\mathcal{B}}(z_i)-\varphi^3_{\mathcal{A}}(z_i)\right)^2}{\varphi^3_{\mathcal{B}}(z_i)+\varphi^3_{\mathcal{A}}(z_i)+2}\right)}\\
&= D_{TD}(\mathcal{B},\mathcal{A})
\end{aligned}$$

Hence, we can obtain $D_{TD}(\mathcal{A},\mathcal{B}) = D_{TD}(\mathcal{B},\mathcal{A})$.

Proof 4 *Given three FFSs $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ on UOD $\mathbb{Z}$. If $\mathcal{A} \subseteq \mathcal{B} \subseteq \mathcal{C}$, we have:*

$$\phi^3_{\mathcal{A}}(z_i) \le \phi^3_{\mathcal{B}}(z_i) \le \phi^3_{\mathcal{C}}(z_i) \text{ and } \varphi^3_{\mathcal{A}}(\mathrm{z_i}) \le \varphi^3_{\mathcal{B}}(\mathrm{z_i}) \le \varphi^3_{\mathcal{C}}(\mathrm{z_i})$$

Thus, we have:

$$\begin{aligned}
D_{TD}(\mathcal{A},\mathcal{B}) &= \sqrt{\frac{3}{2n}\sum_{i=1}^{n}\left(\frac{\left(\phi^3_{\mathcal{A}}(z_i)-\phi^3_{\mathcal{B}}(z_i)\right)^2}{\phi^3_{\mathcal{A}}(z_i)+\phi^3_{\mathcal{B}}(z_i)+2}+\frac{\left(\varphi^3_{\mathcal{A}}(z_i)-\varphi^3_{\mathcal{B}}(z_i)\right)^2}{\varphi^3_{\mathcal{A}}(z_i)+\varphi^3_{\mathcal{B}}(z_i)+2}\right)}\\
&\le \sqrt{\frac{3}{2n}\sum_{i=1}^{n}\left(\frac{\left(\phi^3_{\mathcal{A}}(z_i)-\phi^3_{\mathcal{C}}(z_i)\right)^2}{\phi^3_{\mathcal{A}}(z_i)+\phi^3_{\mathcal{C}}(z_i)+2}+\frac{\left(\varphi^3_{\mathcal{A}}(z_i)-\varphi^3_{\mathcal{C}}(z_i)\right)^2}{\varphi^3_{\mathcal{A}}(z_i)+\varphi^3_{\mathcal{C}}(z_i)+2}\right)}\\
&= D_{PFS}(\mathcal{A},\mathcal{C})
\end{aligned}$$

Similarly, we can get $D_{TD}(\mathcal{B},\mathcal{C}) \le D_{TD}(\mathcal{A},\mathcal{C})$.

Hereafter, we use several numerical examples to show the validity and properties of D_{TD}.

Example 1 *Consider three FFSs $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ in UOD $\mathbb{Z}=\{z_1,z_2\}$. These FFSs are described as follows:*

$$\mathcal{A}=\{\langle z_1,0.8,0.4\rangle,\langle z_2,0.5,0.5\rangle\}$$
$$\mathcal{B}=\{\langle z_1,0.6,0.9\rangle,\langle z_2,0.4,0.7\rangle\}$$
$$\mathcal{C}=\{\langle z_1,0.8,0.4\rangle,\langle z_2,0.5,0.5\rangle\}$$

The D_{TD} between PFSs $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ can be measured as:

$$D_{TD}(\mathcal{A},\mathcal{B})=\sqrt{\frac{3}{2\times 2}\left(\frac{(0.8^3-0.6^3)^2}{0.8^3+0.6^3+2}+\frac{(0.4^3-0.9^3)^2}{0.4^3+0.9^3+2}+\frac{(0.5^3-0.4^3)^2}{0.5^3+0.4^3+2}+\frac{(0.5^3-0.7^3)^2}{0.5^3+0.7^3+2}\right)}=0.3892$$

$$D_{TD}(\mathcal{B},\mathcal{A})=\sqrt{\frac{3}{2\times 2}\left(\frac{(0.6^3-0.8^3)^2}{0.6^3+0.8^3+2}+\frac{(0.9^3-0.4^3)^2}{0.9^3+0.4^3+2}+\frac{(0.4^3-0.5^3)^2}{0.4^3+0.5^3+2}+\frac{(0.7^3-0.5^3)^2}{0.7^3+0.5^3+2}\right)}=0.3892$$

$$D_{TD}(\mathcal{A},\mathcal{C})=\sqrt{\frac{3}{2\times 2}\left(\frac{(0.8^3-0.8^3)^2}{0.8^3+0.8^3+2}+\frac{(0.4^3-0.4^3)^2}{0.4^3+0.4^3+2}+\frac{(0.5^3-0.5^3)^2}{0.5^3+0.5^3+2}+\frac{(0.5^3-0.5^3)^2}{0.5^3+0.5^3+2}\right)}=0$$

According to the above results, we can demonstrate the properties of symmetry and separability of D_{TD}.

Example 2 *Consider three FFSs $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ in UOD $\mathbb{Z}=\{z_1,z_2\}$. These FFSs are described as follows:*

$$\mathcal{A}=\{\langle z_1,0.2,0.9\rangle,\langle z_2,0.3,0.7\rangle\}$$
$$\mathcal{B}=\{\langle z_1,0.5,0.7\rangle,\langle z_2,0.4,0.6\rangle\}$$
$$\mathcal{C}=\{\langle z_1,0.7,0.4\rangle,\langle z_2,0.6,0.4\rangle\}$$

Obviously, $\mathcal{A}\subseteq\mathcal{B}\subseteq\mathcal{C}$. Thus, we have:

$$D_{TD}(\mathcal{A},\mathcal{B})=0.2154,D_{TD}(\mathcal{A},\mathcal{C})=0.4367,D_{TD}(\mathcal{B},\mathcal{C})=0.2322$$

and

$$D_{TD}(\mathcal{A},\mathcal{B})\leq D_{TD}(\mathcal{A},\mathcal{C}),D_{TD}(\mathcal{B},\mathcal{C})\leq D_{TD}(\mathcal{A},\mathcal{C})$$

Hence, we illustrate the property of containment of D_{TD}.

Example 3 *Consider two FFSs $\mathcal{A}$ and $\mathcal{B}$ in UOD $\mathbb{Z}=\{z\}$. These FFSs are described as follows:*

$$\mathcal{A}=\{\langle z,\tau,\rho\rangle\},\mathcal{C}=\{\langle z,\rho,\tau\rangle\}$$

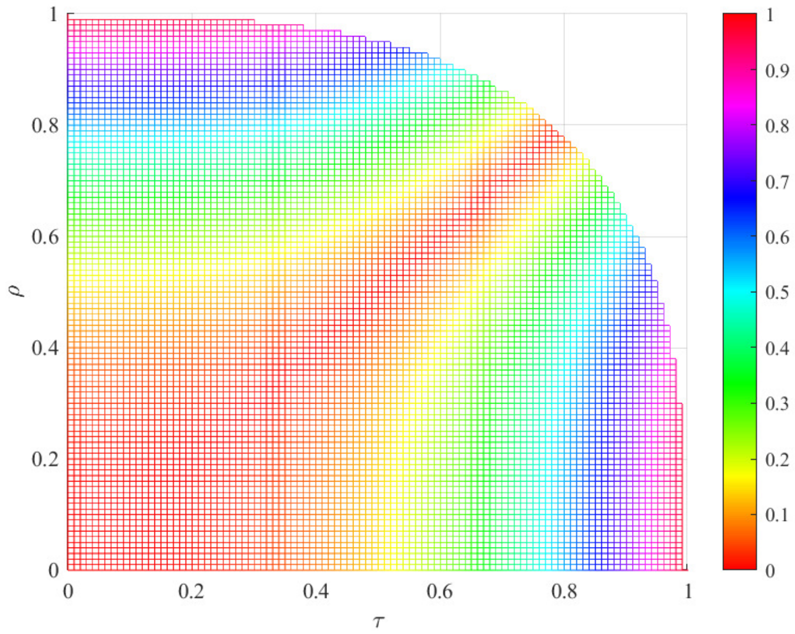

Figure 1: $\tau^3 + \rho^3 \leq 1$.

The parameters τ and ρ are in the range of $[0, 1]$, and satisfy $\tau^3 + \rho^3 \leq 1$, as shown in Figure 1. The D_{TD} between $\mathcal{A}$ and $\mathcal{B}$ can be measured as shown in Figure 2. We can see that as τ and ρ change, D_{TD} always changes between 0 and 1. Therefore, we can verify the property of bounded of D_{TD}.

4. Application in medical diagnosis

Given five kinds of diagnosis $\mathcal{A} = \{\mathcal{A}_1 : (Viral\ fever),\ \mathcal{A}_2 : (Malaria),\ \mathcal{A}_3 : (Typhoid),\ \mathcal{A}_4 : (Stomach\,problem),\ \mathcal{A}_5 : (Chest\,problem)\}$ having the different symptoms is represented in Table 1, and suppose there exists a patient, denoted $\mathcal{B}$. Our aim is to determine the diagnosis of the patients $\mathcal{B}$.

Table 1: FFSs of medical diagnosis problem.

FFS	Symptoms				
	z_1	z_2	z_3	z_4	z_5
$\mathcal{A}_1$	$\langle z_1, 0.4, 0.0\rangle$	$\langle z_2, 0.3, 0.5\rangle$	$\langle z_3, 0.1, 0.7\rangle$	$\langle z_4, 0.4, 0.3\rangle$	$\langle z_5, 0.1, 0.7\rangle$
$\mathcal{A}_2$	$\langle z_1, 0.7, 0.0\rangle$	$\langle z_2, 0.2, 0.6\rangle$	$\langle z_3, 0.0, 0.9\rangle$	$\langle z_4, 0.7, 0.0\rangle$	$\langle z_5, 0.1, 0.8\rangle$
$\mathcal{A}_3$	$\langle z_1, 0.3, 0.3\rangle$	$\langle z_2, 0.6, 0.1\rangle$	$\langle z_3, 0.2, 0.7\rangle$	$\langle z_4, 0.2, 0.6\rangle$	$\langle z_5, 0.1, 0.9\rangle$
$\mathcal{A}_4$	$\langle z_1, 0.1, 0.7\rangle$	$\langle z_2, 0.2, 0.4\rangle$	$\langle z_3, 0.8, 0.0\rangle$	$\langle z_4, 0.2, 0.7\rangle$	$\langle z_5, 0.2, 0.7\rangle$
$\mathcal{A}_5$	$\langle z_1, 0.1, 0.8\rangle$	$\langle z_2, 0.0, 0.8\rangle$	$\langle z_3, 0.2, 0.8\rangle$	$\langle z_4, 0.2, 0.8\rangle$	$\langle z_5, 0.8, 0.1\rangle$
$\mathcal{B}$	$\langle z_1, 0.0, 0.8\rangle$	$\langle z_2, 0.4, 0.4\rangle$	$\langle z_3, 0.6, 0.0\rangle$	$\langle z_4, 0.1, 0.7\rangle$	$\langle z_5, 0.1, 0.8\rangle$

The implementation of the proposed distance measure is expressed below:

Step 1: The distance measure D_{TD} is used to compute the distances between $\mathcal{B}$ and $\mathcal{A}_j(j = 1, 2, 3, 4, 5.)$, the results are shown as:

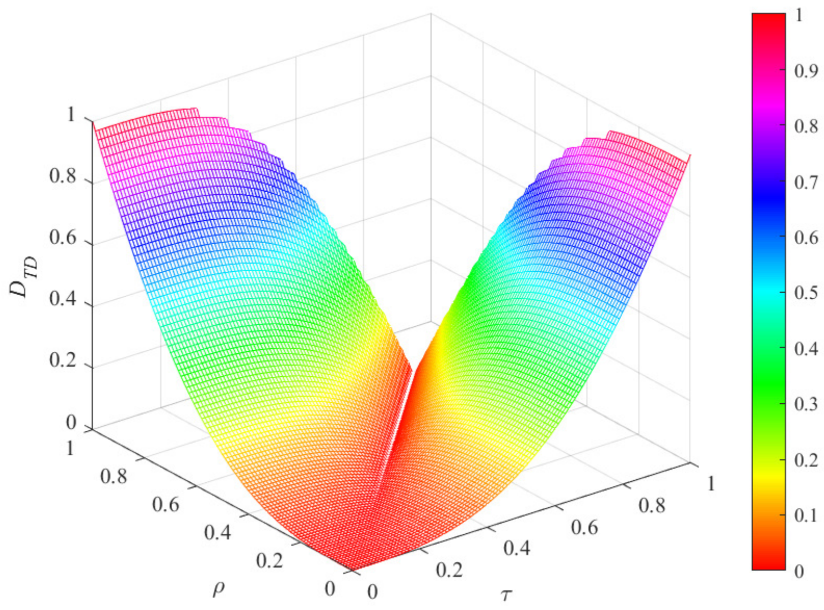

Figure 2: D_{TD} in Example 3.

$$D_{TD}(\mathcal{A}_1,\mathcal{B})=\sqrt{\frac{3}{2\times 5}\left(\begin{array}{c}\frac{(0.4^3-0.0^3)^2}{0.4^3+0.0^3+2}+\frac{(0.0^3-0.8^3)^2}{0.0^3+0.8^3+2}+\frac{(0.3^3-0.4^3)^2}{0.3^3+0.4^3+2}+\frac{(0.5^3-0.4^3)^2}{0.5^3+0.4^3+2}+\frac{(0.1^3-0.6^3)^2}{0.1^3+0.6^3+2}\\+\frac{(0.7^3-0.0^3)^2}{0.7^3+0.0^3+2}+\frac{(0.4^3-0.1^3)^2}{0.4^3+0.1^3+2}+\frac{(0.3^3-0.7^3)^2}{0.3^3+0.7^3+2}+\frac{(0.1^3-0.1^3)^2}{0.1^3+0.1^3+2}+\frac{(0.7^3-0.8^3)^2}{0.7^3+0.8^3+2}\end{array}\right)}=0.2428$$

$$D_{TD}(\mathcal{A}_2,\mathcal{B})=\sqrt{\frac{3}{2\times 5}\left(\begin{array}{c}\frac{(0.7^3-0.0^3)^2}{0.7^3+0.0^3+2}+\frac{(0.0^3-0.8^3)^2}{0.0^3+0.8^3+2}+\frac{(0.2^3-0.4^3)^2}{0.2^3+0.4^3+2}+\frac{(0.6^3-0.4^3)^2}{0.6^3+0.4^3+2}+\frac{(0.0^3-0.6^3)^2}{0.0^3+0.6^3+2}\\+\frac{(0.9^3-0.0^3)^2}{0.9^3+0.0^3+2}+\frac{(0.7^3-0.1^3)^2}{0.7^3+0.1^3+2}+\frac{(0.0^3-0.7^3)^2}{0.0^3+0.7^3+2}+\frac{(0.1^3-0.1^3)^2}{0.1^3+0.1^3+2}+\frac{(0.8^3-0.8^3)^2}{0.8^3+0.8^3+2}\end{array}\right)}=0.3551$$

$$D_{TD}(\mathcal{A}_3,\mathcal{B})=\sqrt{\frac{3}{2\times 5}\left(\begin{array}{c}\frac{(0.3^3-0.0^3)^2}{0.3^3+0.0^3+2}+\frac{(0.3^3-0.8^3)^2}{0.3^3+0.8^3+2}+\frac{(0.6^3-0.4^3)^2}{0.6^3+0.4^3+2}+\frac{(0.1^3-0.4^3)^2}{0.1^3+0.4^3+2}+\frac{(0.2^3-0.6^3)^2}{0.2^3+0.6^3+2}\\+\frac{(0.7^3-0.0^3)^2}{0.7^3+0.0^3+2}+\frac{(0.2^3-0.1^3)^2}{0.2^3+0.1^3+2}+\frac{(0.6^3-0.7^3)^2}{0.6^3+0.7^3+2}+\frac{(0.1^3-0.1^3)^2}{0.1^3+0.1^3+2}+\frac{(0.9^3-0.8^3)^2}{0.9^3+0.8^3+2}\end{array}\right)}=0.3022$$

$$D_{TD}(\mathcal{A}_4,\mathcal{B})=\sqrt{\frac{3}{2\times 5}\left(\begin{array}{c}\frac{(0.1^3-0.0^3)^2}{0.1^3+0.0^3+2}+\frac{(0.7^3-0.8^3)^2}{0.7^3+0.8^3+2}+\frac{(0.2^3-0.4^3)^2}{0.2^3+0.4^3+2}+\frac{(0.4^3-0.4^3)^2}{0.4^3+0.4^3+2}+\frac{(0.8^3-0.6^3)^2}{0.8^3+0.6^3+2}\\+\frac{(0.0^3-0.0^3)^2}{0.0^3+0.0^3+2}+\frac{(0.2^3-0.1^3)^2}{0.2^3+0.1^3+2}+\frac{(0.7^3-0.7^3)^2}{0.7^3+0.7^3+2}+\frac{(0.2^3-0.1^3)^2}{0.2^3+0.1^3+2}+\frac{(0.7^3-0.8^3)^2}{0.7^3+0.8^3+2}\end{array}\right)}=0.1892$$

$$D_{TD}(\mathcal{A}_5,\mathcal{B})=\sqrt{\frac{3}{2\times 5}\left(\begin{array}{c}\frac{(0.1^3-0.0^3)^2}{0.1^3+0.0^3+2}+\frac{(0.8^3-0.8^3)^2}{0.8^3+0.8^3+2}+\frac{(0.0^3-0.4^3)^2}{0.0^3+0.4^3+2}+\frac{(0.8^3-0.4^3)^2}{0.8^3+0.4^3+2}+\frac{(0.2^3-0.6^3)^2}{0.2^3+0.6^3+2}\\+\frac{(0.8^3-0.0^3)^2}{0.8^3+0.0^3+2}+\frac{(0.2^3-0.1^3)^2}{0.2^3+0.1^3+2}+\frac{(0.8^3-0.7^3)^2}{0.8^3+0.7^3+2}+\frac{(0.8^3-0.1^3)^2}{0.8^3+0.1^3+2}+\frac{(0.1^3-0.8^3)^2}{0.1^3+0.8^3+2}\end{array}\right)}=0.3682$$

Step 2: The minimum distance between $\mathcal{B}$ and $\mathcal{A}_j$ is described as follows:

$$D_{TD}(\mathcal{A}_4,\mathcal{B})=0.1892$$

Step 3: The decision result of $\mathcal{B}$ is expressed as follows:

$$\mathcal{A}_4\rightarrow\mathcal{B}$$

From Table 2, we can see that the proposed distance measure D_{TD} diagnose the patient $\mathcal{B}$ as the disease $\mathcal{A}_4$ according to the principle of the minimum distance between FFSs. Whereas, D_T cannot

Table 2: Medical diagnosis results of D_T and the proposed D_{TD}.

Methods	Distance measures				
	$(\mathcal{A}_1, \mathcal{B})$	$(\mathcal{A}_2, \mathcal{B})$	$(\mathcal{A}_3, \mathcal{B})$	$(\mathcal{A}_4, \mathcal{B})$	$(\mathcal{A}_5, \mathcal{B})$
D_T [44]	0.3609	0.4557	0.4275	NaN	0.5043
D_{TD}	0.2428	0.3551	0.3022	0.1892	0.3682

obtain reasonable diagnostic result because its denominator is 0, which makes D_T invalid in some cases.

5. Conclusion

In this paper, we first analyze the serious shortcoming of the distance measure of FFSs in the literature [44], and propose a new distance measure of FFSs based on triangular divergence. Furthermore, we prove that the proposed distance measure satisfies some basic properties and further demonstrate its effectiveness through several numerical examples. Finally, we test the performance of the proposed distance measure and the literature [44] in a medical diagnosis application, and the results show that the proposed distance measure can well overcome the limitation of the literature [44]. In future work, we intend to explore more deeply the potential of the proposed distance measure in other applications.

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Data Availability Statement
The data information is included in this paper.

Conflicts of Interest
The author declares no conflicts of interest.

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