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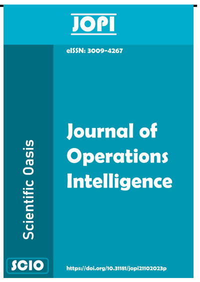

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An Approach to Multi-Attribute Decision Making Based on Spherical Fuzzy Einstein Z-Number Aggregation Information

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ABSTRACT

In this study, we first introduced the spherical fuzzy Z-numbers (SFZNs) and developed some basic operational rules. SFZNs can be used effectively to make true ambiguous judgments, reflecting the fuzzy nature, flexibility, and applicability of decisions making data. Furthermore, We developed some spherical fuzzy Einstein Z-number weighted averaging/geometric aggregation operators and their important axioms. Finally, We developed the algorithms based on the proposed operators to tackle the uncertain information in decision making problems. Finally, developed the relative comparison and discussion analysis to show the practicability of the technique.

1. Introduction

The positive membership degree of a set is discussed in the theory of FSs, which was first presented by Zadeh [1] in 1965. Numerous disciplines, including artificial intelligence, engineering, economics, computer science, and others, have broad applications for FS theory [2]. Additionally, Bellman and Zadeh launched the study of MADM in a fuzzy environment [3] in 1970. It has always been difficult to deal with vague and ambiguous events because they are present in almost every scientific field. In order to achieve this,

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Zadeh presented the idea of a FS, in which he discussed an object's uncertainty or event using a grade of membership m with a value in the range [0,1]. IFS described the uncertainty degree associated with an item using a positive grade ρ that derives its result from an interval that Atanassov [4] presented, is based on two grades, ρ and υ, which stand for an object's degree of positive and negative, respectively. The IFS theory expresses each element by a positive and negative-membership degree, which makes it more effective than the FS theory at dealing with vagueness. Following that, Xu [5] examined a few aggregation operators (AOs) in the IF environment and how they applied to multi-criteria decision-making (MCDM), basing his work on weighted average operator [6] along with ordered weighted average operator [7]. The soft set (SS) theory, developed by Molodtsov [8] in order to tackle the complexity and ambiguity of many issues that appear in the fields of engineering, social science, medical science, economics, and other related fields, is another concept that builds on the FS theory. Maji et al. [9] provided the first example of a practical application of SSs to decision-making issues. In addition, they developed the idea of a fuzzy soft set (FSS), which combines FS and SS, and obtained some of its properties [10]. The SS and FSS theories have numerous important literary applications [11, 12, 13, 14]. The maximum, neutral, and minimum membership degrees of an element in a set are provided by a picture fuzzy set (PFS). These theories have been successfully applied in a variety of different contexts, but when dealing with human opinions that involve more answers of the types yes, no, abstain, and refusal, these approaches are insufficient. The idea of a PFS, which is an extension of IFS, was thus proposed by Cuong [15]. For instance, if someone indicated that they had a yes vote of 0.7, a 0.3 abstention, and a no vote of 0.5 regarding the matter, we would get the result $0.7 + 0.5 + 0.3 \nleq 1$. Although Cuong's creation of picture fuzzy sets is best known, decision-makers are somewhat constrained in determining degrees due to the constraints on $\rho(m); \sigma(m)$; and $\upsilon(m)$. Sometimes the total of their membership degrees exceeds 1. In this case, PFS fails to produce a reasonable result. We will use this circumstance as an example in contradiction of the membership degrees: the choices are, in order, (3/5), (1/5), and (3/5). This makes up for the fact that their sum exceeds 1, and PFS cannot handle this kind of data. To address these issues, Ashraf [16] presented an innovative structure by establishing spherical fuzzy sets (SFSs), that expand the space for membership levels $\rho(m); \sigma(m)$; and $\upsilon(m)$ to a somewhat larger extent by satisfying the condition that $0 \leq \rho(m) + \sigma(m) + \upsilon(m) \leq 1$. In comparison to past notions, this structure is significantly more in accordance with human nature, making it one of the most active areas of study today. A versatile mathematical concept, the SFS, has played a pivotal role in generating numerous significant findings across diverse domains. Within the context of decision-making, it has facilitated the development of more resilient and nuanced decision-making processes [17]. The integration of SFSs [18], has notably improved the accuracy and applicability of cosine similarity measures. Furthermore, novel applications of Dombi operators [19], have been harnessed to enhance decision-making processes, with a special emphasis on the crucial task of diagnosing COVID-19 [20]. The synergy of the GRA method [21], with the distinct attributes of the spherical linguistic fuzzy Choquet integral has resulted in the creation of more precise and effective tools. Additionally, the field of infectious disease identification has witnessed a significant breakthrough. This is attributed to the incorporation of complex probabilistic hesitant fuzzy N-soft information [22]. After that, Jin et al. [23] suggested an alternative approach to solving the SFSs, which can be helpful in solving fuzzy multi-criteria group decision-making problems. Gijndogdu and Kahraman expanded the VIKOR, WASPAS, and TOPSIS methods to the spherical fuzzy environment [24, 25, 26]. As several writers examine Einstein interactive AOs of TSFSs, studies have also been done on Einstein AOs based on Einstein t-norms in [27], intuitionistic fuzzy (IF) Einstein geometric AOs are investigated in [28], Riaz et al. [29] explored fuzzy Einstein AOs of Q-rung orthopair. Zhang and Yu explored geometric Choquet AOs with Einstein t-norms under IF environment in [30]. Hamacher AOs based on entropy are developed in [31] under IF structure, Ullah et al. are researching for evaluating the effectiveness of the search and rescue robots by employing T-Spherical Fuzzy Hamacher AOs in [32]. Jana et al. [33] created Picture fuzzy Hamacher AOs which based on Entropy. Wang et al. [34] created researching the use of Pythagorean fuzzy Hamacher Power AOs for assessing business performance.

Aside from the Z-Number (Zadeh, 2011) [35], none of these techniques can, however, accurately reflect the information's reliability. Fuzzy numbers order pair (Γ, Ω), there Γ depicts the limitation information & Ω represents the likelihood that something will be reliable Γ, together, form a Z-Number. Zadeh first proposed the idea of Z-numbers. For instance, Aliev and Alizadeh (2015) [36] proposed a calculation model for DZ-Numbers by combining probabilistic and probabilistic distributions. Following that, Aliev and Huseynov (2016) [37] CDZN on the basis of discrete Z-numbers and suggested continuous Z-number arithmetic operations. Z-Numbers express the restriction Γ and the certainty Ω using a single linguistic expression, such as "the air quality is good, very certain." In this study, in order to accurately represent human cognition. The generalized Z-description numbers are not simply more in line with how people think that the classical Z-Number, but it is also better able to capture the uncertainty present in problem-solving situations. Making decisions is an essential part of everyday life.

DM [38] is the technique of choosing the finest choice from a variety of alternatives. This last phase of the planning process is essential. It becomes a challenging process if you are forced to choose from several excellent possibilities. High performance and high-quality outcomes are only feasible in practice if the research community focuses on overcoming theoretical knowledge gaps and practitioners apply the most recent advancements in their applications to tackle real-world issues. As a result, Spherical fuzzy Einstein Z-Numbers must be introduced to draw on the most recent developments in fuzzy sets, systems, and DM, as well as the related significant business applications. The major goal of this research is to develop the foundation for a new model, the spherical fuzzy Einstein Z-Number model, which is incredibly adept at expressing ambiguous information. It can be used as a useful tool for making actual uncertain decisions, improving the accuracy of the information used to make decisions, and reflecting their fuzziness, flexibility, and applicability.

The main goal of this manuscript is to define the spherical fuzzy Einstein Z-number AOs (SFEZNAOs) and create a concept for these operators in order to address multiple attribute group DM issues. The following is the sectioning of the manuscript: In Section , we review some fundamental ideas and pertinent concepts from the main portion. The spherical fuzzy Einstein Z-Number aggregation operators were introduced in section 3. Then, the Spherical Fuzzy Einstein Z-Number Weighted Averaging (SFEZNWA) and Spherical Fuzzy Einstein Z-Number Weighted Geometric (SFEZNWG) operators are shown in section 4. These operators were made with Spherical Fuzzy Einstein Z-Numbers (SFEZNs), Einstein sum, product, and scalar multiplication. We also examine several fundamental properties of these operators. In Section 5, using a spherical fuzzy Z-Number (SFZN) environment, we create a model to address many attribute groups' DM issues. We give a relative comparison in Section 6. In Section 7, we conclude the results.

2. Preliminaries

Some fundamental definitions for the set M are provided. These fundamental ideas will make it easier for readers to comprehend the suggested work.

2.1 Definition:

On Set M, a spherical fuzzy Z number is defined as

$$Q = \{m, \rho(\Gamma, \Omega)(m), \sigma(\Gamma, \Omega)(m), \upsilon(\Gamma, \Omega)(m) \,|m \ \in M\},$$

where $\rho, \sigma, \upsilon : M \to [0,1]$ are a membership, are a function of membership, abstention, and non-membership, with the condition

$$0 \leq \rho^2(m), \sigma^2(m), \upsilon^2(m) \leq 1,$$

$\forall m \in M$

such that $\rho\left(\Gamma,\Omega\right)(m)=\left(\rho_{\Gamma}(m),\rho_{\Omega}(m)\right)$, $\sigma(\Gamma,\Omega)(m)=(\sigma_{\Gamma}(m),\sigma_{\Omega}(m))$ and $\gamma(\Gamma,\Omega)(m)=(\sigma_{\Gamma}(m),\sigma_{\Omega}(m))$ such that $\rho\left(\Gamma,\Omega\right)(m), \sigma\left(\Gamma,\Omega\right)(m), \upsilon\left(\Gamma,\Omega\right)(m): M\rightarrow[0,1]$ are the order pair of positive, neutral and negative membership grades in a Set M

$$0\leq(\rho_{\Gamma}(m))^2+(\sigma_{\Gamma}(m))^2+(\sigma_{\Gamma}(m))^2\leq 1$$

and

$$0\leq(\rho_{\Omega}(m))^2+(\sigma_{\Omega}(m))^2+(\sigma_{\Omega}(m))^2\leq 1.$$

The element for the standard representation is $[m,\rho\left(\Gamma,\Omega\right)(m),\sigma\left(\Gamma,\Omega\right)(m),\upsilon\left(\Gamma,\Omega\right)(m)]$ in

$S_Q=\left(\rho\left(\Gamma,\Omega\right),\sigma\left(\Gamma,\Omega\right),\upsilon\left(\Gamma,\Omega\right)\right)=\left[(\rho_{\Gamma}(m),\rho_{\Omega}(m)),(\sigma_{\Gamma}(m),\sigma_{\Omega}(m)),(\upsilon_{\Gamma}(m),\upsilon_{\Omega}(m))\right]$ which is named as SFZN.

$t(m)=\sqrt[2]{1-\left(\rho^2\left(m\right)+\sigma^2\left(m\right)+\upsilon^2\left(m\right)\right)}$ is is called the denial degree of m in Q, and (ρ,σ,υ) is known as spherical fuzzy Einstein Z number (SFEZN).

2.2 Definition:

Let $J_1=\left\{\rho_1\left(\Gamma,\Omega\right)(m),\sigma_1\left(\Gamma,\Omega\right)(m),\upsilon_1\left(\Gamma,\Omega\right)(m)\right\}$ and $J_2=\left\{\rho_2\left(\Gamma,\Omega\right)(m),\sigma_2\left(\Gamma,\Omega\right)(m),\upsilon_2\left(\Gamma,\Omega\right)(m)\right\}$. The following are several operations are as described:

$J_1\otimes J_2=\left\langle\rho_1\rho_2\left(\Gamma,\Omega\right),\sigma_1\sigma_2\left(\Gamma,\Omega\right),\sqrt{\upsilon_1^2\left(\Gamma,\Omega\right)+\upsilon_2^2\left(\Gamma,\Omega\right)-\upsilon_1^2\upsilon_2^2\left(\Gamma,\Omega\right)}\right\rangle;$

$J_1\oplus J_2=\left\langle\sqrt{\rho_1^2\left(\Gamma,\Omega\right)+\rho_2^2\left(\Gamma,\Omega\right)-\rho_1^2\rho_2^2\left(\Gamma,\Omega\right)},\sigma_1\sigma_2\left(\Gamma,\Omega\right),\upsilon_1\upsilon_2\left(\Gamma,\Omega\right)\right\rangle;$

$J_1^{\Psi}=\left\langle\left(\rho_1\left(\Gamma,\Omega\right)\right)^{\Psi},\left(\sigma_1\left(\Gamma,\Omega\right)\right)^{\Psi},\sqrt{1-\left(1-\upsilon_1^2\left(\Gamma,\Omega\right)\right)^{\Psi}}\right\rangle;\Psi>0,$

$\Psi J_1=\left\langle\sqrt{1-\left(1-\rho_1^2\left(\Gamma,\Omega\right)\right)^{\Psi}},\left(\sigma_1\left(\Gamma,\Omega\right)\right)^{\Psi},\left(\upsilon_1\left(\Gamma,\Omega\right)\right)^{\Psi}\right\rangle.$

3. Einstein Operational Laws for Spherical fuzzy Einstein Z-Numbers

In this section, we will become familiar with generalized union and intersection for spherical fuzzy Z-Numbers.

$$\begin{aligned} J_1\vee J_2 &= \left\{\rho_1\vee\rho_2\left(\Gamma,\Omega\right),\sigma_1\wedge\sigma_2\left(\Gamma,\Omega\right),\upsilon_1\wedge\upsilon_2\left(\Gamma,\Omega\right)\right\}, \\ J_1\wedge J_2 &= \left\{\rho_1\wedge\rho_2\left(\Gamma,\Omega\right),\sigma_1\wedge\sigma_2\left(\Gamma,\Omega\right),\upsilon_1\vee\upsilon_2\left(\Gamma,\Omega\right)\right\}. \end{aligned}$$

Additionally, we could compose:
$J_1\vee J_2=(\rho_{\Gamma1}\ \vee\rho_{\Gamma2},\rho_{\Omega1}\vee\rho_{\Omega2}),(\sigma_{\Gamma1}\wedge\sigma_{\Gamma2},\sigma_{\Omega1}\wedge\sigma_{\Omega2}),(\upsilon_{\Gamma1}\wedge\upsilon_{\Gamma2},\upsilon_{\Omega1}\wedge\upsilon_{\Omega2});$

$J_1\wedge J_2=(\rho_{\Gamma1}\ \wedge\rho_{\Gamma2},\rho_{\Omega1}\wedge\rho_{\Omega2}),(\sigma_{\Gamma1}\wedge\sigma_{\Gamma2},\sigma_{\Omega1}\wedge\sigma_{\Omega2}),(\upsilon_{\Gamma1}\vee\upsilon_{\Gamma2},\upsilon_{\Omega1}\vee\upsilon_{\Omega2})$

3.1 Definition:

Einstein Operations for Spherical Fuzzy Z-Numbers present Einstein procedures have some drawbacks, and they occasionally fail.Therefore, we suggest On the basis of Einstein operations, we created a few new aggregation operators.

Let $J_1=\left\{\rho_1\left(\Gamma,\Omega\right)(m),\sigma_1\left(\Gamma,\Omega\right)(m),\upsilon_1\left(\Gamma,\Omega\right)(m)\right\}$ and
$J_2=\left\{\rho_2\left(\Gamma,\Omega\right)(m),\sigma_2\left(\Gamma,\Omega\right)(m),\upsilon_2\left(\Gamma,\Omega\right)(m)\right\}$.
Then
$J_1\otimes J_2=\left(\frac{\sqrt{\rho_1^2+\rho_2^2}}{\sqrt{1+\rho_1^2\rho_2^2}},\frac{\sigma_1.\sigma_2}{\sqrt{1+\left(1-\sigma_1^2\right).\left(1-\sigma_2^2\right)}},\frac{\upsilon_1.\upsilon_2}{\sqrt{1+\left(1-\upsilon_1^2\right).\left(1-\upsilon_2^2\right)}}\right);$

$$J_1 \oplus J_2 = \left(\frac{\rho_1 . \rho_2}{\sqrt{1+\left(1-\rho_1^2\right).\left(1-\rho_2^2\right)}}, \frac{\sigma_1 . \sigma_2}{\sqrt{1+\left(1-\sigma_1^2\right).\left(1-\sigma_2^2\right)}}, \frac{\sqrt{\upsilon_1^2+\upsilon_2^2}}{\sqrt{1+\upsilon_1^2 \upsilon_2^2}} \right);$$

$$\Psi J_1 = \left(\frac{\sqrt{\left(1+\rho_1^2\right)^{\Psi}-\left(1-\rho_1^2\right)^{\Psi}}}{\sqrt{\left(1+\rho_1^2\right)^{\Psi}+\left(1-\rho_1^2\right)^{\Psi}}}, \frac{\sqrt{2}(\sigma_1)^{\Psi}}{\sqrt{(2-\sigma_1)^{\Psi}+(\sigma_1)^{\Psi}}}, \frac{\sqrt{2}(\upsilon_1)^{\Psi}}{\sqrt{(2-\upsilon_1)^{\Psi}+(\upsilon_1)^{\Psi}}} \right); \Psi > 0,$$

$$(J_1)^{\Psi} = \left(\frac{\sqrt{2}(\rho_1)^{\Psi}}{\sqrt{(2-\rho_1)^{\Psi}+(\rho_1)^{\Psi}}}, \frac{\sqrt{2}(\sigma_1)^{\Psi}}{\sqrt{(2-\sigma_1)^{\Psi}+(\sigma_1)^{\Psi}}}, \frac{\sqrt{\left(1+\upsilon_1^2\right)^{\Psi}-\left(1-\upsilon_1^2\right)^{\Psi}}}{\sqrt{\left(1+\upsilon_1^2\right)^{\Psi}+\left(1-\upsilon_1^2\right)^{\Psi}}} \right); \Psi > 0.$$

3.2 Definition:

Assume $Q = (\rho\,(\Gamma,\Omega)\,(m), \sigma\,(\Gamma,\Omega)\,(m), \upsilon\,(\Gamma,\Omega)\,(m)) \in$ spherical fuzzy Einstein Z number set. To compare SFEZNs
$Q_r = \langle (m)\,(\rho\,(\Gamma,\Omega)\,, \sigma\,(\Gamma,\Omega)\,, \upsilon\,(\Gamma,\Omega)) \rangle = \langle (\rho_{\Gamma}(m), \rho_{\Omega}(m)), (\sigma_{\Gamma}(m), \sigma_{\Omega}(m))\,, (\sigma_{\Gamma}(m), \sigma_{\Omega}(m)) \rangle\,.$
We introduce the score function as follows:

$$S_J = \frac{1}{3}\left(2 + (\rho_{\Gamma} \cdot \rho_{\Omega}) - (\sigma_{\Gamma} \cdot \sigma_{\Omega}) - (\upsilon_{\Gamma} \cdot \upsilon_{\Omega})\right), \text{ for } Q_r \in [0,1].$$

A SFEZN that receives a higher score is better than others. The supremacy of any two SFZNs will be determined using their accuracy values if their scores are identical. The number that has the highest accuracy value will be better than the others. When precision values are equal, the numbers are regarded as being similar.

4. Spherical fuzzy Einstein Z-Number Aggregation Operators

Here we discussed operators in relation to recently introduced Einstein operations. Additionally, the habitat of spherical fuzzy Einstein Z numbers is covered, along with some of its fundamental properties like monotonicity, boundedness, and idempotency. The efficacy of the suggested work is assessed using an illustration.

4.1 Definition:

In relation to any set of SFEZNA let

$Q_r = (Q = (\rho\,(\Gamma,\Omega)\,(m), \sigma\,(\Gamma,\Omega)\,(m), \upsilon\,(\Gamma,\Omega)\,(m)))\,(r = 1, 2, 3, ..., u)\,.$

The establishment

$SFEZNA\,(S_{Q_1}, S_{Q_2}, S_{Q_3}, ...S_{Q_u}) = \oplus_{r=1}^{u} \Psi_r Q_{ri}$

$$= \left(\begin{array}{c} \left(\frac{\sqrt{\left(1+\rho_{\Gamma i}^2\right)^{\Psi}-\left(1-\rho_{\Gamma i}^2\right)^{\Psi}}}{\sqrt{\left(1+\rho_{\Gamma i}^2\right)^{\Psi}+\left(1-\rho_{\Gamma i}^2\right)^{\Psi}}}, \frac{\sqrt{\left(1+\rho_{\Omega i}^2\right)^{\Psi}-\left(1-\rho_{\Omega i}^2\right)^{\Psi}}}{\sqrt{\left(1+\rho_{\Omega i}^2\right)^{\Psi}+\left(1-\rho_{\Omega i}^2\right)^{\Psi}}} \right), \\ \left(\frac{\sqrt{2}(\sigma_{\Gamma i})^{\Psi}}{\sqrt{(2-\sigma_{\Gamma i})^{\Psi}+(\sigma_{\Gamma i})^{\Psi}}}, \frac{\sqrt{2}(\sigma_{\Omega i})^{\Psi}}{\sqrt{(2-\sigma_{\Omega i})^{\Psi}+(\sigma_{\Omega i})^{\Psi}}} \right), \\ \left(\frac{\sqrt{2}(\upsilon_{\Gamma i})^{\Psi}}{\sqrt{(2-\upsilon_{\Gamma i})^{\Psi}+(\upsilon_{\Gamma i})^{\Psi}}}, \frac{\sqrt{2}(\upsilon_{\Omega i})^{\Psi}}{\sqrt{(2-\upsilon_{\Omega i})^{\Psi}+(\upsilon_{\Omega i})^{\Psi}}} \right) \end{array} \right),$$

is described as the Spherical fuzzy Einstein Z-Number Averaging operator (SFEZNA), where

$\Psi_r = (\Psi_1, \Psi_2, ..., \Psi_u)^{\Gamma}$is the weight vector of Q_r with $\Psi_r \in [0,1]$ and

$\sum\limits_{r=1}^{u} \Psi_r = 1$

4.2 Theorem:

Let $Q = (\rho(\Gamma,\Omega)(m), \sigma(\Gamma,\Omega)(m), \upsilon(\Gamma,\Omega)(m)) \in SFEZN$ and weight vectors of Q_r $(r \in M)$ be $\Psi_r = (\Psi_1, \Psi_2, ..., \Psi_u)^{\Gamma}$ related to limit $\sum\limits_{r=1}^{u} \Psi_r = 1$. Then the operators of the SFEZWA is mapping of the $Q^u \to Q$ such that

SFEZNWA $(Q_1, Q_2, Q_3, ..., Q_n) = \oplus_{r=1}^{u} \Psi_r Q_{ri}$

$$= \left(\begin{array}{c} \left(\dfrac{\sqrt{\prod\limits_{r=1}^{u}\left(1+\rho_{\Gamma i}^2\right)^{\Psi} - \prod\limits_{r=1}^{u}\left(1-\rho_{\Gamma i}^2\right)^{\Psi}}}{\sqrt{\prod\limits_{r=1}^{u}\left(1+\rho_{\Gamma i}^2\right)^{\Psi} + \prod\limits_{r=1}^{u}\left(1-\rho_{\Gamma i}^2\right)^{\Psi}}}, \dfrac{\sqrt{\prod\limits_{r=1}^{u}\left(1+\rho_{\Omega i}^2\right)^{\Psi} - \prod\limits_{r=1}^{u}\left(1-\rho_{\Omega i}^2\right)^{\Psi}}}{\sqrt{\prod\limits_{r=1}^{u}\left(1+\rho_{\Omega i}^2\right)^{\Psi} + \prod\limits_{r=1}^{u}\left(1-\rho_{\Omega i}^2\right)^{\Psi}}} \right), \\ \left(\dfrac{\sqrt{2} \prod\limits_{r=1}^{u}(\sigma_{\Gamma i})^{\Psi}}{\sqrt{\prod\limits_{r=1}^{u}(2-\sigma_{\Gamma i})^{\Psi} + \prod\limits_{r=1}^{u}(\sigma_{\Gamma i})^{\Psi}}}, \dfrac{\sqrt{2} \prod\limits_{r=1}^{u}(\sigma_{\Omega i})^{\Psi}}{\sqrt{\prod\limits_{r=1}^{u}(2-\sigma_{\Omega i})^{\Psi} + \prod\limits_{r=1}^{u}(\sigma_{\Omega i})^{\Psi}}} \right), \\ \left(\dfrac{\sqrt{2} \prod\limits_{r=1}^{u}(\upsilon_{\Gamma i})^{\Psi}}{\sqrt{\prod\limits_{r=1}^{u}(2-\upsilon_{\Gamma i})^{\Psi} + \prod\limits_{r=1}^{u}(\upsilon_{\Gamma i})^{\Psi}}}, \dfrac{\sqrt{2} \prod\limits_{r=1}^{u}(\upsilon_{\Omega i})^{\Psi}}{\sqrt{\prod\limits_{r=1}^{u}(2-\upsilon_{\Omega i})^{\Psi} + \prod\limits_{r=1}^{u}(\upsilon_{\Omega i})^{\Psi}}} \right) \end{array} \right).$$

Prof:

By Mathematical Induction to prove the Equation ,

$r = 2$

$SFEZNWA(Q_1, Q_2) = \sum_{r=1}^{2} \Psi_r Q_r = \Psi_1 Q_1 + \Psi_2 Q_2$

According to we have,

$$\Psi_1 \cdot Q_1 = \left(\begin{array}{c} \left(\dfrac{\sqrt{\left(1+\rho_{\Gamma 1}^2\right)^{\Psi_1} - \left(1-\rho_{\Gamma 1}^2\right)^{\Psi_1}}}{\sqrt{\left(1+\rho_{\Gamma 1}^2\right)^{\Psi_1} + \left(1-\rho_{\Gamma 1}^2\right)^{\Psi_1}}}, \dfrac{\sqrt{\left(1+\rho_{\Omega 1}^2\right)^{\Psi_1} - \left(1-\rho_{\Omega 1}^2\right)^{\Psi_1}}}{\sqrt{\left(1+\rho_{\Omega 1}^2\right)^{\Psi_1} + \left(1-\rho_{\Omega 1}^2\right)^{\Psi_1}}} \right), \\ \left(\dfrac{\sqrt{2}(\sigma_{\Gamma 1})^{\Psi_1}}{\sqrt{(2-\sigma_{\Gamma 1})^{\Psi_1} + (\sigma_{\Gamma 1})^{\Psi_1}}}, \dfrac{\sqrt{2}(\sigma_{\Omega 1})^{\Psi_1}}{\sqrt{(2-\sigma_{\Omega 1})^{\Psi_1} + (\sigma_{\Omega 1})^{\Psi_1}}} \right), \\ \left(\dfrac{\sqrt{2}(\upsilon_{\Gamma 1})^{\Psi_1}}{\sqrt{(2-\upsilon_{\Gamma 1})^{\Psi_1} + (\upsilon_{\Gamma 1})^{\Psi_1}}}, \dfrac{\sqrt{2}(\upsilon_{\Omega 1})^{\Psi_1}}{\sqrt{(2-\upsilon_{\Omega 1})^{\Psi_1} + (\upsilon_{\Omega 1})^{\Psi_1}}} \right) \end{array} \right),$$

$$\Psi_2 \cdot Q_2 = \left(\begin{array}{c} \left(\dfrac{\sqrt{\left(1+\rho_{\Gamma 2}^2\right)^{\Psi_2} - \left(1-\rho_{\Gamma 2}^2\right)^{\Psi_2}}}{\sqrt{\left(1+\rho_{\Gamma 2}^2\right)^{\Psi_2} + \left(1-\rho_{\Gamma 2}^2\right)^{\Psi_2}}}, \dfrac{\sqrt{\left(1+\rho_{\Omega 2}^2\right)^{\Psi_2} - \left(1-\rho_{\Omega 2}^2\right)^{\Psi_2}}}{\sqrt{\left(1+\rho_{\Omega 2}^2\right)^{\Psi_2} + \left(1-\rho_{\Omega 2}^2\right)^{\Psi_2}}} \right), \\ \left(\dfrac{\sqrt{2}\left(\sigma_{\Gamma 2}\right)^{\Psi_2}}{\sqrt{(2-\sigma_{\Gamma 2})^{\Psi_2} + (\sigma_{\Gamma 2})^{\Psi_2}}}, \dfrac{\sqrt{2}(\sigma_{\Omega 2})^{\Psi_2}}{\sqrt{(2-\sigma_{\Omega 2})^{\Psi_2} + (\sigma_{\Omega 2})^{\Psi_2}}} \right), \\ \left(\dfrac{\sqrt{2}(\upsilon_{\Gamma 2})^{\Psi_2}}{\sqrt{(2-\upsilon_{\Gamma 2})^{\Psi_2} + (\upsilon_{\Gamma 2})^{\Psi_2}}}, \dfrac{\sqrt{2}(\upsilon_{\Omega 2})^{\Psi_2}}{\sqrt{(2-\upsilon_{\Omega 2})^{\Psi_2} + (\upsilon_{\Omega 2})^{\Psi_2}}} \right) \end{array} \right).$$

Then, $SFEZNWA(Q_1, Q_2) = \Psi_1 Q_1 + \Psi_2 Q_2$

$$
\left[\begin{array}{c}
\left(\begin{array}{c}
\frac{\sqrt{\frac{\left(1+\rho_{\Gamma1}^2\right)^{\Psi_1}-\left(1-\rho_{\Gamma1}^2\right)^{\Psi_1}}{\sqrt{\left(1+\rho_{\Gamma1}^2\right)^{\Psi_1}+\left(1-\rho_{\Gamma1}^2\right)^{\Psi_1}}}+\frac{\left(1+\rho_{\Gamma2}^2\right)^{\Psi_2}-\left(1-\rho_{\Gamma2}^2\right)^{\Psi_2}}{\left(1+\rho_{\Gamma2}^2\right)^{\Psi_2}+\left(1-\rho_{\Gamma2}^2\right)^{\Psi_2}}}}{\sqrt{1+\frac{\left(1+\rho_{\Gamma1}^2\right)^{\Psi_1}-\left(1-\rho_{\Gamma1}^2\right)^{\Psi_1}}{\sqrt{\left(1+\rho_{\Gamma1}^2\right)^{\Psi_1}+\left(1-\rho_{\Gamma1}^2\right)^{\Psi_1}}}\cdot\frac{\left(1+\rho_{\Gamma2}^2\right)^{\Psi_2}-\left(1-\rho_{\Gamma2}^2\right)^{\Psi_2}}{\left(1+\rho_{\Gamma2}^2\right)^{\Psi_2}+\left(1-\rho_{\Gamma2}^2\right)^{\Psi_2}}}}, \\
\frac{\sqrt{\frac{\left(1+\rho_{\Omega1}^2\right)^{\Psi_1}-\left(1-\rho_{\Omega1}^2\right)^{\Psi_1}}{\sqrt{\left(1+\rho_{\Omega1}^2\right)^{\Psi_1}+\left(1-\rho_{\Omega1}^2\right)^{\Psi_1}}}+\frac{\left(1+\rho_{\Omega2}^2\right)^{\Psi_2}-\left(1-\rho_{\Omega2}^2\right)^{\Psi_2}}{\left(1+\rho_{\Omega2}^2\right)^{\Psi_2}+\left(1-\rho_{\Omega2}^2\right)^{\Psi_2}}}}{\sqrt{1+\frac{\left(1+\rho_{\Omega1}^2\right)^{\Psi_1}-\left(1-\rho_{\Omega1}^2\right)^{\Psi_1}}{\sqrt{\left(1+\rho_{\Omega1}^2\right)^{\Psi_1}+\left(1-\rho_{\Omega1}^2\right)^{\Psi_1}}}\cdot\frac{\left(1+\rho_{\Omega2}^2\right)^{\Psi_2}-\left(1-\rho_{\Omega2}^2\right)^{\Psi_2}}{\left(1+\rho_{\Omega2}^2\right)^{\Psi_2}+\left(1-\rho_{\Omega2}^2\right)^{\Psi_2}}}}
\end{array}\right), \\
\left(\begin{array}{c}
\frac{\frac{\sqrt{2}\left(\sigma_{\Gamma1}\right)^{\Psi_1}}{\sqrt{\left(2-\sigma_{\Gamma1}^2\right)^{\Psi_1}+\left(\sigma_{\Gamma1}^2\right)^{\Psi_1}}}\cdot\frac{\sqrt{2}\left(\sigma_{\Gamma2}\right)^{\Psi_2}}{\sqrt{\left(2-\sigma_{\Gamma2}^2\right)^{\Psi_2}+\left(\sigma_{\Gamma2}^2\right)^{\Psi_2}}}}{\sqrt{1+\left(1-\left(\frac{\sqrt{2}\left(\sigma_{\Gamma1}\right)^{\Psi_1}}{\sqrt{\left(2-\sigma_{\Gamma1}^2\right)^{\Psi_1}+\left(\sigma_{\Gamma1}^2\right)^{\Psi_1}}}\right)^2\right)\cdot\left(1-\left(\frac{\sqrt{2}\left(\sigma_{\Gamma2}\right)^{\Psi_2}}{\sqrt{\left(2-\sigma_{\Gamma2}^2\right)^{\Psi_2}+\left(\sigma_{\Gamma2}^2\right)^{\Psi_2}}}\right)^2\right)}}, \\
\frac{\frac{\sqrt{2}\left(\sigma_{\Omega1}\right)^{\Psi_1}}{\sqrt{\left(2-\sigma_{\Omega1}^2\right)^{\Psi_1}+\left(\sigma_{\Omega1}^2\right)^{\Psi_1}}}\cdot\frac{\sqrt{2}\left(\sigma_{\Omega2}\right)^{\Psi_2}}{\sqrt{\left(2-\sigma_{\Omega2}^2\right)^{\Psi_2}+\left(\sigma_{\Omega2}^2\right)^{\Psi_2}}}}{\sqrt{1+\left(1-\left(\frac{\sqrt{2}\left(\sigma_{\Omega1}\right)^{\Psi_1}}{\sqrt{\left(2-\sigma_{\Omega1}^2\right)^{\Psi_1}+\left(\sigma_{\Omega1}^2\right)^{\Psi_1}}}\right)^2\right)\cdot\left(1-\left(\frac{\sqrt{2}\left(\sigma_{\Omega2}\right)^{\Psi_2}}{\sqrt{\left(2-\sigma_{\Omega2}^2\right)^{\Psi_2}+\left(\sigma_{\Omega2}^2\right)^{\Psi_2}}}\right)^2\right)}}
\end{array}\right), \\
\left(\begin{array}{c}
\frac{\frac{\sqrt{2}\left(\upsilon_{\Gamma1}\right)^{\Psi_1}}{\sqrt{\left(2-\upsilon_{\Gamma1}^2\right)^{\Psi_1}+\left(\upsilon_{\Gamma1}^2\right)^{\Psi_1}}}\cdot\frac{\sqrt{2}\left(\upsilon_{\Gamma2}\right)^{\Psi_2}}{\sqrt{\left(2-\upsilon_{\Gamma2}^2\right)^{\Psi_2}+\left(\upsilon_{\Gamma2}^2\right)^{\Psi_2}}}}{\sqrt{1+\left(1-\left(\frac{\sqrt{2}\left(\upsilon_{\Gamma1}\right)^{\Psi_1}}{\sqrt{\left(2-\upsilon_{\Gamma1}^2\right)^{\Psi_1}+\left(\upsilon_{\Gamma1}^2\right)^{\Psi_1}}}\right)^2\right)\cdot\left(1-\left(\frac{\sqrt{2}\left(\upsilon_{\Gamma1}\right)^{\Psi_2}}{\sqrt{\left(2-\upsilon_{\Gamma2}^2\right)^{\Psi_2}+\left(\upsilon_{\Gamma2}^2\right)^{\Psi_2}}}\right)^2\right)}}, \\
\frac{\frac{\sqrt{2}\left(\upsilon_{\Omega1}\right)^{\Psi_1}}{\sqrt{\left(2-\upsilon_{\Omega1}^2\right)^{\Psi_1}+\left(\upsilon_{\Omega1}^2\right)^{\Psi_1}}}\cdot\frac{\sqrt{2}\left(\upsilon_{\Omega2}\right)^{\Psi_2}}{\sqrt{\left(2-\upsilon_{\Omega2}^2\right)^{\Psi_2}+\left(\upsilon_{\Omega2}^2\right)^{\Psi_2}}}}{\sqrt{1+\left(1-\left(\frac{\sqrt{2}\left(\upsilon_{\Omega1}\right)^{\Psi_1}}{\sqrt{\left(2-\upsilon_{\Omega1}^2\right)^{\Psi_1}+\left(\upsilon_{\Omega1}^2\right)^{\Psi_1}}}\right)^2\right)\cdot\left(1-\left(\frac{\sqrt{2}\left(\upsilon_{\Omega1}\right)^{\Psi_2}}{\sqrt{\left(2-\upsilon_{\Omega2}^2\right)^{\Psi_2}+\left(\upsilon_{\Omega2}^2\right)^{\Psi_2}}}\right)^2\right)}}
\end{array}\right)
\end{array}\right]
$$

$$
= \left[\begin{array}{c}
\left(\sqrt{\frac{\left(1+\rho_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(1+\rho_{\Gamma2}^2\right)^{\Psi_2}-\left(1-\rho_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(1-\rho_{\Gamma2}^2\right)^{\Psi_2}}{\left(1+\rho_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(1+\rho_{\Gamma2}^2\right)^{\Psi_2}+\left(1-\rho_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(1-\rho_{\Gamma2}^2\right)^{\Psi_2}}}, \sqrt{\frac{\left(1+\rho_{\Omega1}^2\right)^{\Psi_1}\cdot\left(1+\rho_{\Omega2}^2\right)^{\Psi_2}-\left(1-\rho_{\Omega1}^2\right)^{\Psi_1}\cdot\left(1-\rho_{\Omega2}^2\right)^{\Psi_2}}{\left(1+\rho_{\Omega1}^2\right)^{\Psi_1}\cdot\left(1+\rho_{\Omega2}^2\right)^{\Psi_2}+\left(1-\rho_{\Omega1}^2\right)^{\Psi_1}\cdot\left(1-\rho_{\Omega2}^2\right)^{\Psi_2}}}\right), \\
\left(\frac{\sqrt{2}\left(\sigma_{\Gamma1}\cdot\sigma_{\Gamma2}\right)}{\sqrt{\left(2-\sigma_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(2-\sigma_{\Gamma2}^2\right)^{\Psi_2}+\left(\sigma_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(\sigma_{\Gamma2}^2\right)^{\Psi_2}}}, \frac{\sqrt{2}\left(\sigma_{\Omega1}\cdot\sigma_{\Omega2}\right)}{\sqrt{\left(2-\sigma_{\Omega1}^2\right)^{\Psi_1}\cdot\left(2-\sigma_{\Omega2}^2\right)^{\Psi_2}+\left(\sigma_{\Omega1}^2\right)^{\Psi_1}\cdot\left(\sigma_{\Omega2}^2\right)^{\Psi_2}}}\right), \\
\left(\frac{\sqrt{2}\left(\upsilon_{\Gamma1}\cdot\upsilon_{\Gamma2}\right)}{\sqrt{\left(2-\upsilon_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(2-\upsilon_{\Gamma2}^2\right)^{\Psi_2}+\left(\upsilon_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(\upsilon_{\Gamma2}^2\right)^{\Psi_2}}}, \frac{\sqrt{2}\left(\upsilon_{\Omega1}\cdot\upsilon_{\Omega2}\right)}{\sqrt{\left(2-\upsilon_{\Omega1}^2\right)^{\Psi_1}\cdot\left(2-\upsilon_{\Omega2}^2\right)^{\Psi_2}+\left(\upsilon_{\Omega1}^2\right)^{\Psi_1}\cdot\left(\upsilon_{\Omega2}^2\right)^{\Psi_2}}}\right)
\end{array}\right].
$$

Thus the Equation , is true for $r = 2$.

Consider that Equation , is true for $r = q$, we have,

$$SFEZNWA\left(Q_1,Q_2,...,Q_q\right)=\left[\begin{array}{c}\left(\frac{\sqrt{\prod_{r=1}^{q}\left(1+\rho_{\Gamma r}^2\right)^{\Psi_r}-\prod_{r=1}^{q}\left(1+\rho_{\Gamma r}^2\right)^{\Psi_r}}}{\sqrt{\prod_{r=1}^{q}\left(1+\rho_{\Gamma r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(1+\rho_{\Gamma r}^2\right)^{\Psi_r}}},\frac{\sqrt{\prod_{r=1}^{q}\left(1+\rho_{\Omega r}^2\right)^{\Psi_r}-\prod_{r=1}^{q}\left(1+\rho_{\Omega r}^2\right)^{\Psi_r}}}{\sqrt{\prod_{r=1}^{q}\left(1+\rho_{\Omega r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(1+\rho_{\Omega r}^2\right)^{\Psi_r}}}\right),\\ \left(\frac{\sqrt{2}\prod_{r=1}^{q}\left(\sigma_{\Gamma r}\right)^{\Psi_r}}{\sqrt{\prod_{r=1}^{q}\left(2-\sigma_{\Gamma r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(\sigma_{\Gamma r}^2\right)^{\Psi_r}}},\frac{\sqrt{2}\prod_{r=1}^{q}\left(\sigma_{\Omega r}\right)^{\Psi_r}}{\sqrt{\prod_{r=1}^{q}\left(2-\sigma_{\Omega r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(\sigma_{\Omega r}^2\right)^{\Psi_r}}}\right),\\ \left(\frac{\sqrt{2}\prod_{r=1}^{q}\left(\upsilon_{\Gamma r}\right)^{\Psi_r}}{\sqrt{\prod_{r=1}^{q}\left(2-\upsilon_{\Gamma r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(\upsilon_{\Gamma r}^2\right)^{\Psi_r}}},\frac{\sqrt{2}\prod_{r=1}^{q}\left(\upsilon_{\Omega r}\right)^{\Psi_r}}{\sqrt{\prod_{r=1}^{q}\left(2-\upsilon_{\Omega r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(\upsilon_{\Omega r}^2\right)^{\Psi_r}}}\right)\end{array}\right]$$

The next step is to demonstrate that is true for $r=q+1$. To this end, we have
$SFEZNWA\left(Q_1,Q_2,Q_3,...,Q_q,Q_{q+1}\right)=\sum_{r=1}^{z}\Psi_rQ_r+\Psi_{q+1}Q_{q+1}$

$$=\left[\begin{array}{c}\left(\frac{\sqrt{\prod_{r=1}^{q}\left(1+\rho_{\Gamma r}^2\right)^{\Psi_r}-\prod_{r=1}^{q}\left(1-\rho_{\Gamma r}^2\right)^{\Psi_r}}}{\sqrt{\prod_{r=1}^{q}\left(1+\rho_{\Gamma r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(1-\rho_{\Gamma r}^2\right)^{\Psi_r}}},\frac{\sqrt{\prod_{r=1}^{q}\left(1+\rho_{\Omega r}^2\right)^{\Psi_r}-\prod_{r=1}^{q}\left(1-\rho_{\Omega r}^2\right)^{\Psi_r}}}{\sqrt{\prod_{r=1}^{q}\left(1+\rho_{\Omega r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(1-\rho_{\Omega r}^2\right)^{\Psi_r}}}\right),\\ \left(\frac{\sqrt{2}\prod_{r=1}^{q}\left(\sigma_{\Gamma r}\right)^{\Psi_r}}{\sqrt{\prod_{r=1}^{q}\left(2-\sigma_{\Gamma r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(\sigma_{\Gamma r}^2\right)^{\Psi_r}}},\frac{\sqrt{2}\prod_{r=1}^{q}\left(\sigma_{\Omega r}\right)^{\Psi_r}}{\sqrt{\prod_{r=1}^{q}\left(2-\sigma_{\Omega r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(\sigma_{\Omega r}^2\right)^{\Psi_r}}}\right),\\ \left(\frac{\sqrt{2}\prod_{r=1}^{q}\left(\upsilon_{\Gamma r}\right)^{\Psi_r}}{\sqrt{\prod_{r=1}^{q}\left(2-\upsilon_{\Gamma r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(\upsilon_{\Gamma r}^2\right)^{\Psi_r}}},\frac{\sqrt{2}\prod_{r=1}^{q}\left(\upsilon_{\Omega r}\right)^{\Psi_r}}{\sqrt{\prod_{r=1}^{q}\left(2-\upsilon_{\Omega r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(\upsilon_{\Omega r}^2\right)^{\Psi_r}}}\right)\end{array}\right]$$

$$+\left[\begin{array}{c}\left(\frac{\sqrt{\left(1+\rho_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}-\left(1-\rho_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}}}{\sqrt{\left(1+\rho_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}+\left(1-\rho_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}}},\frac{\sqrt{\left(1+\rho_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}-\left(1-\rho_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}}}{\sqrt{\left(1+\rho_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}+\left(1-\rho_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}}}\right),\\ \left(\frac{\sqrt{2}\left(\sigma_{\Gamma(q+1)}\right)^{\Psi_{q+1}}}{\sqrt{\left(2-\sigma_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}+\left(\sigma_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}}},\frac{\sqrt{2}\left(\sigma_{\Omega(q+1)}\right)^{\Psi_{q+1}}}{\sqrt{\left(2-\sigma_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}+\left(\sigma_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}}}\right),\\ \left(\frac{\sqrt{2}\left(\upsilon_{\Gamma(q+1)}\right)^{\Psi_{q+1}}}{\sqrt{\left(2-\upsilon_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}+\left(\upsilon_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}}},\frac{\sqrt{2}\left(\upsilon_{\Omega(q+1)}\right)^{\Psi_{q+1}}}{\sqrt{\left(2-\upsilon_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}+\left(\upsilon_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}}}\right)\end{array}\right]$$

$$= \left[\begin{array}{c} \left(\frac{\sqrt{\prod\limits_{r=1}^{q}\left(1+\rho_{\Gamma r}^{2}\right)^{\Psi_r}-\prod\limits_{r=1}^{q}\left(1-\rho_{\Gamma r}^{2}\right)^{\Psi_r}}}{\sqrt{\prod\limits_{r=1}^{q}\left(1+\rho_{\Gamma r}^{2}\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(1-\rho_{\Gamma r}^{2}\right)^{\Psi_r}}}, \frac{\sqrt{\prod\limits_{r=1}^{q}\left(1+\rho_{\Omega r}^{2}\right)^{\Psi_r}-\prod\limits_{r=1}^{q}\left(1-\rho_{\Omega r}^{2}\right)^{\Psi_r}}}{\sqrt{\prod\limits_{r=1}^{q}\left(1+\rho_{\Omega r}^{2}\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(1-\rho_{\Omega r}^{2}\right)^{\Psi_r}}} \right), \\ \left(\frac{\sqrt{2}\prod\limits_{r=1}^{q}\left(\sigma_{\Gamma r}\right)^{\Psi_r}}{\sqrt{\prod\limits_{r=1}^{q}\left(2-\sigma_{\Gamma r}^{2}\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(\sigma_{\Gamma r}^{2}\right)^{\Psi_r}}}, \frac{\sqrt{2}\prod\limits_{r=1}^{q}\left(\sigma_{\Omega r}\right)^{\Psi_r}}{\sqrt{\prod\limits_{r=1}^{q}\left(2-\sigma_{\Omega r}^{2}\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(\sigma_{\Omega r}^{2}\right)^{\Psi_r}}} \right), \\ \left(\frac{\sqrt{2}\prod\limits_{r=1}^{q}\left(\upsilon_{\Gamma r}\right)^{\Psi_r}}{\sqrt{\prod\limits_{r=1}^{q}\left(2-\upsilon_{\Gamma r}^{2}\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(\upsilon_{\Gamma r}^{2}\right)^{\Psi_r}}}, \frac{\sqrt{2}\prod\limits_{r=1}^{q}\left(\upsilon_{\Omega r}\right)^{\Psi_r}}{\sqrt{\prod\limits_{r=1}^{q}\left(2-\upsilon_{\Omega r}^{2}\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(\upsilon_{\Omega r}^{2}\right)^{\Psi_r}}} \right) \end{array} \right].$$

Consequently, it holds for r= z + 1. The result is valid for all n according to the principle of mathematical induction.

The following SFEZNWA operators can all be proven to be true quite easily by using Definitions.

4.3 Theorem:

1)Let $Q_r = \{\rho\left(\Gamma,\Omega\right)(m), \sigma\left(\Gamma,\Omega\right)(m), \upsilon\left(\Gamma,\Omega\right)(m)\} \in SFEZN$ where $r = (1, 2, 3, ..., u)$, if $Q_1 = Q_2 = ...Q_{u-1} = Q_u = Q$, then $SFEZNWA(Q_1, Q_2, Q_3, ..., Q_u) = Q$.

2)Let $Q_r = \{\rho\left(\Gamma,\Omega\right)(m), \sigma\left(\Gamma,\Omega\right)(m), \upsilon\left(\Gamma,\Omega\right)(m)\} \in SFEZN$ where $r = (1, 2, 3, ..., u)$ and $Q^{+} = \max Q_r$,$Q^{-} = \min Q_r$. where $Q^{-} \leq SFEZNWA(Q_1, Q_2, Q_3, ..., Q_u) \leq Q^{+}$.

3)Let $Q_r = \{\rho\left(\Gamma,\Omega\right)(m), \sigma\left(\Gamma,\Omega\right)(m), \upsilon\left(\Gamma,\Omega\right)(m)\}$ and $Q_r^{\sim} = \left\{\rho^{\sim}\left(\Gamma,\Omega\right)(m), \sigma^{\sim}\left(\Gamma,\Omega\right)(m), \upsilon^{\sim}\left(\Gamma,\Omega\right)(m)\right\} \in SFEZNWA\left(Q, Q^{\sim} \in M\right)$

$i.e.Q_r \leq Q_r^{\sim} \forall r$.Then,$SFEZNWA(Q_1, Q_2, Q_3, ..., Q_u) \leq SFEZNWA(Q_1^{\sim}, Q_2^{\sim}, Q_3^{\sim}, ..., Q_u^{\sim})$.

4.4 Definition:

Let $Q_r = \{\rho\left(\Gamma,\Omega\right)(m), \sigma\left(\Gamma,\Omega\right)(m), \upsilon\left(\Gamma,\Omega\right)(m)\}\left(r \in M\right)$. The For SFZN(O), the Spherical Fuzzy Einstein Ordered Weighted Averaging Operator (SFEZNOWA) is described as $SFEZNOWA(Q_1, Q_2, ..., Q_u) = \sum\limits_{r=1}^{u} \Psi_r Q_{\Phi(r)}$,

where $\Phi\left(r\right)$ represent Order and $\left(\Phi\left(1\right), \Phi\left(2\right), ..., \Phi\left(u\right)\right)$ is a permutation of $\left(1, 2, ..., u\right)$, subject to $Q_{\Phi(r-1)} \geq Q_{\Phi(r)} \forall r$. Also $\Psi_r\left(r \in M\right)$ is the weight vector with $\Psi_r \geq 0$, $\sum\limits_{r=1}^{u} \Psi_r = 1$.

4.5 Theorem:

Let $Q_r = \{\rho\left(\Gamma,\Omega\right)(m), \sigma\left(\Gamma,\Omega\right)(m), \upsilon\left(\Gamma,\Omega\right)(m)\}\left(r \in M\right)$.The and the $\Psi_r\left(r \in M\right)$ is the weight vector with $\Psi_r \geq 0$, $\sum\limits_{r=1}^{u} \Psi_r = 1$.The SFEZNOWA is a mapping $D^u \to D$ defined as:

$$SFEZNOWA\left(Q_1, Q_2, Q_3, ..., Q_u\right) = \sum\limits_{r=1}^{u} \Psi_r Q_{\Phi(r)}$$

$$= \left(\begin{array}{c} \left(\frac{\sqrt{\prod_{r=1}^{u}\left(1+\rho^2_{\Phi(\Gamma i)}\right)^{\Psi} - \prod_{r=1}^{u}\left(1-\rho^2_{\Phi(\Gamma i)}\right)^{\Psi}}}{\sqrt{\prod_{r=1}^{u}\left(1+\rho^2_{\Phi(\Gamma i)}\right)^{\Psi} + \prod_{r=1}^{u}\left(1-\rho^2_{\Phi(\Gamma i)}\right)^{\Psi}}}, \frac{\sqrt{\prod_{r=1}^{u}\left(1+\rho^2_{\Phi(\Omega i)}\right)^{\Psi} - \prod_{r=1}^{u}\left(1-\rho^2_{\Phi(\Omega i)}\right)^{\Psi}}}{\sqrt{\prod_{r=1}^{u}\left(1+\rho^2_{\Phi(\Omega i)}\right)^{\Psi} + \prod_{r=1}^{u}\left(1-\rho^2_{\Phi(\Omega i)}\right)^{\Psi}}} \right), \\ \left(\frac{\sqrt{2}\ \prod_{r=1}^{u}\left(\sigma_{\Phi(\Gamma i)}\right)^{\Psi}}{\sqrt{\prod_{r=1}^{u}\left(2-\sigma_{\Phi(\Gamma i)}\right)^{\Psi} + \prod_{r=1}^{u}\left(\sigma_{\Phi(\Gamma i)}\right)^{\Psi}}}, \frac{\sqrt{2}\ \prod_{r=1}^{u}\left(\sigma_{\Phi(\Omega i)}\right)^{\Psi}}{\sqrt{\prod_{r=1}^{u}\left(2-\sigma_{\Phi(\Omega i)}\right)^{\Psi} + \prod_{r=1}^{u}\left(\sigma_{\Phi(\Omega i)}\right)^{\Psi}}} \right), \\ \left(\frac{\sqrt{2}\ \prod_{r=1}^{u}\left(\upsilon_{\Phi(\Gamma i)}\right)^{\Psi}}{\sqrt{\prod_{r=1}^{u}\left(2-\upsilon_{\Phi(\Gamma i)}\right)^{\Psi} + \prod_{r=1}^{u}\left(\upsilon_{\Phi(\Gamma i)}\right)^{\Psi}}}, \frac{\sqrt{2}\ \prod_{r=1}^{u}\left(\upsilon_{\Phi(\Omega i)}\right)^{\Psi}}{\sqrt{\prod_{r=1}^{u}\left(2-\upsilon_{\Phi(\Omega i)}\right)^{\Psi} + \prod_{r=1}^{u}\left(\upsilon_{\Phi(\Omega i)}\right)^{\Psi}}} \right) \end{array} \right)$$

where $\Phi(r)$ is denoted for Ordered and $(\Phi(1), \Phi(2), \Phi(3), ..., \Phi(u))$ is a permutation of $(1, 2, 3, ..., u)$, subject to $Q_{\Phi(r-1)} \geq Q_{\Phi(r)}$ for all r.

Proof: Since the proof of the theorem is the same as that of the theorem , it is not included here.

The following SFEZNOWA operators can all be proven to be true quite easily by using Definitions.

1)Let $Q_r = \{\rho(\Gamma,\Omega)(m), \sigma(\Gamma,\Omega)(m), \upsilon(\Gamma,\Omega)(m)\} \in SFEZN$ where $r = (1, 2, 3, ..., u)$, if

$Q_1 = Q_2 = ...Q_{u-1} = Q_u = Q$, then $SFEZNOWA(Q_1, Q_2, Q_3, ..., Q_u) = Q$.

2)Let $Q_r = \{\rho(\Gamma,\Omega)(m), \sigma(\Gamma,\Omega)(m), \upsilon(\Gamma,\Omega)(m)\} \in SFEZN$ where $r = (1, 2, 3, ..., u)$ and $Q^+ = \max Q_r$

,$Q^- = \min Q_r$. where $Q^- \leq SFEZNOWA(Q_1, Q_2, Q_3, ..., Q_u) \leq Q^+$.

3)Let $Q_r = \{\rho(\Gamma,\Omega)(m), \sigma(\Gamma,\Omega)(m), \upsilon(\Gamma,\Omega)(m)\}$ and

$Q_r^{\tilde{}} = \left\{\rho^{\tilde{}}(\Gamma,\Omega)(m), \sigma^{\tilde{}}(\Gamma,\Omega)(m), \upsilon^{\tilde{}}(\Gamma,\Omega)(m)\right\} \in SFEZNOWA\left(Q, Q^{\tilde{}} \in M\right)$

$i.e. Q_r \leq Q_r^{\tilde{}} \forall r.$Then,$SFEZNOWA(Q_1, Q_2, Q_3, ..., Q_u) \leq SFEZNOWA(Q_1^{\tilde{}}, Q_2^{\tilde{}}, Q_3^{\tilde{}}, ..., Q_u^{\tilde{}})$.

4.6 Definition:

Let $Q_r = (Q = (\rho(\Gamma,\Omega)(m), \sigma(\Gamma,\Omega)(m), \upsilon(\Gamma,\Omega)(m)))\ (r = 1, 2, 3, ..., u)$.

The Einstein weighted geometric operator is characterized as $SFEZNA(S_{Q_1}, S_{Q_2}, S_{Q_3}, ...S_{Q_u}) = \prod_{r=1}^{u} Q_{ri}^{\Psi_r}$

is described as the Spherical fuzzy Einstein Z-Number Geometric operator (SFEZNG), where $\Psi_r = (\Psi_1, \Psi_2, ..., \Psi_u)^{\Gamma}$is the weight vector $\Psi_r \geq 0$, $\sum_{r=1}^{u} \Psi_r = 1$

4.7 Theorem:

Let $Q = (\rho(\Gamma,\Omega)(m), \sigma(\Gamma,\Omega)(m), \upsilon(\Gamma,\Omega)(m)) \in SFEZN$ and weight vectors of Q_r $(r \in M)$ be $\Psi_r = (\Psi_1, \Psi_2, ..., \Psi_u)^{\Gamma}$ related to limit $\sum_{r=1}^{u} \Psi_r = 1$.Then the operators of the SFEZWG is mapping of the $Q^u \to Q$ such that

SFEZWG $(Q_1, Q_2, Q_3, ..., Q_n) = \oplus_{r=1}^{u} Q_{ri}^{\Psi_r}$

$$= \left(\begin{array}{c} \left(\frac{\sqrt{2} \quad \prod_{r=1}^{u} (\rho_{\Gamma i})^{\Psi r}}{\sqrt{\prod_{r=1}^{u} (2-\rho_{\Gamma i})^{\Psi r} + \prod_{r=1}^{u} (\rho_{\Gamma i})^{\Psi r}}}, \frac{\sqrt{2} \quad \prod_{r=1}^{u} (\rho_{\Omega i})^{\Psi r}}{\sqrt{\prod_{r=1}^{u} (2-\rho_{\Omega i})^{\Psi r} + \prod_{r=1}^{u} (\rho_{\Omega i})^{\Psi r}}} \right), \\ \left(\frac{\sqrt{2} \quad \prod_{r=1}^{u} (\sigma_{\Gamma i})^{\Psi r}}{\sqrt{\prod_{r=1}^{u} (2-\sigma_{\Gamma i})^{\Psi r} + (\sigma_{\Gamma i})^{\Psi r}}}, \frac{\sqrt{2} \quad \prod_{r=1}^{u} (\sigma_{\Omega i})^{\Psi r}}{\sqrt{\prod_{r=1}^{u} (2-\sigma_{\Omega i})^{\Psi r} + \prod_{r=1}^{u} (\sigma_{\Omega i})^{\Psi r}}} \right), \\ \left(\frac{\sqrt{\prod_{r=1}^{u} \left(1+\upsilon_{\Gamma i}^{2}\right)^{\Psi r} - \prod_{r=1}^{u} \left(1-\upsilon_{\Gamma i}^{2}\right)^{\Psi r}}}{\sqrt{\prod_{r=1}^{u} \left(1+\upsilon_{\Gamma i}^{2}\right)^{\Psi r} + \prod_{r=1}^{u} \left(1-\upsilon_{\Gamma i}^{2}\right)^{\Psi r}}}, \frac{\sqrt{\prod_{r=1}^{u} \left(1+\upsilon_{\Omega i}^{2}\right)^{\Psi r} - \prod_{r=1}^{u} \left(1-\upsilon_{\Omega i}^{2}\right)^{\Psi r}}}{\sqrt{\prod_{r=1}^{u} \left(1+\upsilon_{\Omega i}^{2}\right)^{\Psi r} + \prod_{r=1}^{u} \left(1-\upsilon_{\Omega i}^{2}\right)^{\Psi r}}} \right) \end{array} \right).$$

Proof:

By Mathematical Induction r to prove the Equation ,

$r = 2$

$SFEZNG\left(Q_1, Q_2\right) = \sum_{r=1}^{2} Q_{ri}^{\Psi r} = Q_1^{\Psi_1} + Q_2^{\Psi_2}$

According to we have,

$$Q_1^{\Psi_1} = \left(\begin{array}{c} \left(\frac{\sqrt{\left(1+\rho_{\Gamma 1}^{2}\right)^{\Psi} - \left(1-\rho_{\Gamma 1}^{2}\right)^{\Psi}}}{\sqrt{\left(1+\rho_{\Gamma 1}^{2}\right)^{\Psi} + \left(1-\rho_{\Gamma 1}^{2}\right)^{\Psi}}}, \frac{\sqrt{\left(1+\rho_{\Omega 1}^{2}\right)^{\Psi} - \left(1-\rho_{\Omega 1}^{2}\right)^{\Psi}}}{\sqrt{\left(1+\rho_{\Omega 1}^{2}\right)^{\Psi} + \left(1-\rho_{\Omega 1}^{2}\right)^{\Psi}}} \right), \\ \left(\frac{\sqrt{2}(\sigma_{\Gamma 1})^{\Psi}}{\sqrt{(2-\sigma_{\Gamma 1})^{\Psi} + (\sigma_{\Gamma 1})^{\Psi}}}, \frac{\sqrt{2}(\sigma_{\Omega 1})^{\Psi}}{\sqrt{(2-\sigma_{\Omega 1})^{\Psi} + (\sigma_{\Omega 1})^{\Psi}}} \right), \\ \left(\frac{\sqrt{2}(\upsilon_{\Gamma 1})^{\Psi}}{\sqrt{(2-\upsilon_{\Gamma 1})^{\Psi} + (\upsilon_{\Gamma 1})^{\Psi}}}, \frac{\sqrt{2}(\upsilon_{\Omega 1})^{\Psi}}{\sqrt{(2-\upsilon_{\Omega 1})^{\Psi} + (\upsilon_{\Omega 1})^{\Psi}}} \right) \end{array} \right),$$

$$Q_2^{\Psi_2} = \left(\begin{array}{c} \left(\frac{\sqrt{\left(1+\rho_{\Gamma 2}^{2}\right)^{\Psi} - \left(1-\rho_{\Gamma 2}^{2}\right)^{\Psi}}}{\sqrt{\left(1+\rho_{\Gamma 2}^{2}\right)^{\Psi} + \left(1-\rho_{\Gamma 2}^{2}\right)^{\Psi}}}, \frac{\sqrt{\left(1+\rho_{\Omega 2}^{2}\right)^{\Psi} - \left(1-\rho_{\Omega 2}^{2}\right)^{\Psi}}}{\sqrt{\left(1+\rho_{\Omega 2}^{2}\right)^{\Psi} + \left(1-\rho_{\Omega 2}^{2}\right)^{\Psi}}} \right), \\ \left(\frac{\sqrt{2}\left(\sigma_{\Gamma 2}\right)^{\Psi}}{\sqrt{(2-\sigma_{\Gamma 2})^{\Psi} + (\sigma_{\Gamma 2})^{\Psi}}}, \frac{\sqrt{2}(\sigma_{\Omega 2})^{\Psi}}{\sqrt{(2-\sigma_{\Omega 2})^{\Psi} + (\sigma_{\Omega 2})^{\Psi}}} \right), \\ \left(\frac{\sqrt{2}(\upsilon_{\Gamma 2})^{\Psi}}{\sqrt{(2-\upsilon_{\Gamma 2})^{\Psi} + (\upsilon_{\Gamma 2})^{\Psi}}}, \frac{\sqrt{2}(\upsilon_{\Omega 2})^{\Psi}}{\sqrt{(2-\upsilon_{\Omega 2})^{\Psi} + (\upsilon_{\Omega 2})^{\Psi}}} \right) \end{array} \right).$$

Then, $SFEZNG\left(Q_1, Q_2\right) = \Psi_1 Q_1 + \Psi_2 Q_2$

$=$

$$
\begin{bmatrix}
\left(\dfrac{\frac{\sqrt{2}(\rho_{\Gamma1})^{\Psi_1}}{\sqrt{\left(2-\rho_{\Gamma1}^2\right)^{\Psi_1}+\left(\rho_{\Gamma1}^2\right)^{\Psi_1}}}\cdot\frac{\sqrt{2}(\rho_{\Gamma2})^{\Psi_2}}{\sqrt{\left(2-\rho_{\Gamma2}^2\right)^{\Psi_2}+\left(\rho_{\Gamma2}^2\right)^{\Psi_2}}}}{\sqrt{1+\left(1-\left(\frac{\sqrt{2}(\rho_{\Gamma1})^{\Psi_1}}{\sqrt{\left(2-\rho_{\Gamma1}^2\right)^{\Psi_1}+\left(\rho_{\Gamma1}^2\right)^{\Psi_1}}}\right)^2\right)\cdot\left(1-\left(\frac{\sqrt{2}(\rho_{\Gamma1})^{\Psi_2}}{\sqrt{\left(2-\rho_{\Gamma2}^2\right)^{\Psi_2}+\left(\rho_{\Gamma2}^2\right)^{\Psi_2}}}\right)^2\right)}},
\dfrac{\frac{\sqrt{2}(\rho_{\Omega1})^{\Psi_1}}{\sqrt{\left(2-\rho_{\Omega1}^2\right)^{\Psi_1}+\left(\rho_{\Omega1}^2\right)^{\Psi_1}}}\cdot\frac{\sqrt{2}(\rho_{\Omega2})^{\Psi_2}}{\sqrt{\left(2-\rho_{\Omega2}^2\right)^{\Psi_2}+\left(\rho_{\Omega2}^2\right)^{\Psi_2}}}}{\sqrt{1+\left(1-\left(\frac{\sqrt{2}(\rho_{\Omega1})^{\Psi_1}}{\sqrt{\left(2-\rho_{\Omega1}^2\right)^{\Psi_1}+\left(\rho_{\Omega1}^2\right)^{\Psi_1}}}\right)^2\right)\cdot\left(1-\left(\frac{\sqrt{2}(\rho_{\Omega1})^{\Psi_2}}{\sqrt{\left(2-\rho_{\Omega2}^2\right)^{\Psi_2}+\left(\rho_{\Omega2}^2\right)^{\Psi_2}}}\right)^2\right)}}\right), \\
\left(\dfrac{\frac{\sqrt{2}(\sigma_{\Gamma1})^{\Psi_1}}{\sqrt{\left(2-\sigma_{\Gamma1}^2\right)^{\Psi_1}+\left(\sigma_{\Gamma1}^2\right)^{\Psi_1}}}\cdot\frac{\sqrt{2}(\sigma_{\Gamma2})^{\Psi_2}}{\sqrt{\left(2-\sigma_{\Gamma2}^2\right)^{\Psi_2}+\left(\sigma_{\Gamma2}^2\right)^{\Psi_2}}}}{\sqrt{1+\left(1-\left(\frac{\sqrt{2}(\sigma_{\Gamma1})^{\Psi_1}}{\sqrt{\left(2-\sigma_{\Gamma1}^2\right)^{\Psi_1}+\left(\sigma_{\Gamma1}^2\right)^{\Psi_1}}}\right)^2\right)\cdot\left(1-\left(\frac{\sqrt{2}(\sigma_{\Gamma2})^{\Psi_2}}{\sqrt{\left(2-\sigma_{\Gamma2}^2\right)^{\Psi_2}+\left(\sigma_{\Gamma2}^2\right)^{\Psi_2}}}\right)^2\right)}},
\dfrac{\frac{\sqrt{2}(\sigma_{\Omega1})^{\Psi_1}}{\sqrt{\left(2-\sigma_{\Omega1}^2\right)^{\Psi_1}+\left(\sigma_{\Omega1}^2\right)^{\Psi_1}}}\cdot\frac{\sqrt{2}(\sigma_{\Omega2})^{\Psi_2}}{\sqrt{\left(2-\sigma_{\Omega2}^2\right)^{\Psi_2}+\left(\sigma_{\Omega2}^2\right)^{\Psi_2}}}}{\sqrt{1+\left(1-\left(\frac{\sqrt{2}(\sigma_{\Omega1})^{\Psi_1}}{\sqrt{\left(2-\sigma_{\Omega1}^2\right)^{\Psi_1}+\left(\sigma_{\Omega1}^2\right)^{\Psi_1}}}\right)^2\right)\cdot\left(1-\left(\frac{\sqrt{2}(\sigma_{\Omega2})^{\Psi_2}}{\sqrt{\left(2-\sigma_{\Omega2}^2\right)^{\Psi_2}+\left(\sigma_{\Omega2}^2\right)^{\Psi_2}}}\right)^2\right)}}\right), \\
\left(\dfrac{\sqrt{\frac{\left(1+\upsilon_{\Gamma1}^2\right)^{\Psi_1}-\left(1-\upsilon_{\Gamma1}^2\right)^{\Psi_1}}{\sqrt{\left(1+\upsilon_{\Gamma1}^2\right)^{\Psi_1}+\left(1-\upsilon_{\Gamma1}^2\right)^{\Psi_1}}}+\frac{\left(1+\upsilon_{\Gamma2}^2\right)^{\Psi_2}-\left(1-\upsilon_{\Gamma2}^2\right)^{\Psi_2}}{\left(1+\upsilon_{\Gamma2}^2\right)^{\Psi_2}+\left(1-\upsilon_{\Gamma2}^2\right)^{\Psi_2}}}}{\sqrt{1+\frac{\left(1+\upsilon_{\Gamma1}^2\right)^{\Psi_1}-\left(1-\upsilon_{\Gamma1}^2\right)^{\Psi_1}}{\sqrt{\left(1+\upsilon_{\Gamma1}^2\right)^{\Psi_1}+\left(1-\upsilon_{\Gamma1}^2\right)^{\Psi_1}}}\cdot\frac{\left(1+\upsilon_{\Gamma2}^2\right)^{\Psi_2}-\left(1-\upsilon_{\Gamma2}^2\right)^{\Psi_2}}{\left(1+\upsilon_{\Gamma2}^2\right)^{\Psi_2}+\left(1-\upsilon_{\Gamma2}^2\right)^{\Psi_2}}}},
\dfrac{\sqrt{\frac{\left(1+\upsilon_{\Omega1}^2\right)^{\Psi_1}-\left(1-\upsilon_{\Omega1}^2\right)^{\Psi_1}}{\sqrt{\left(1+\upsilon_{\Omega1}^2\right)^{\Psi_1}+\left(1-\upsilon_{\Omega1}^2\right)^{\Psi_1}}}+\frac{\left(1+\upsilon_{\Omega2}^2\right)^{\Psi_2}-\left(1-\upsilon_{\Omega2}^2\right)^{\Psi_2}}{\left(1+\upsilon_{\Omega2}^2\right)^{\Psi_2}+\left(1-\upsilon_{\Omega2}^2\right)^{\Psi_2}}}}{\sqrt{1+\frac{\left(1+\upsilon_{\Omega1}^2\right)^{\Psi_1}-\left(1-\upsilon_{\Omega1}^2\right)^{\Psi_1}}{\sqrt{\left(1+\upsilon_{\Omega1}^2\right)^{\Psi_1}+\left(1-\upsilon_{\Omega1}^2\right)^{\Psi_1}}}\cdot\frac{\left(1+\upsilon_{\Omega2}^2\right)^{\Psi_2}-\left(1-\upsilon_{\Omega2}^2\right)^{\Psi_2}}{\left(1+\upsilon_{\Omega2}^2\right)^{\Psi_2}+\left(1-\upsilon_{\Omega2}^2\right)^{\Psi_2}}}}\right)
\end{bmatrix}
$$

$$
=\begin{bmatrix}
\left(\dfrac{\sqrt{2}(\rho_{\Gamma1}\cdot\rho_{\Gamma2})}{\sqrt{\left(2-\rho_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(2-\rho_{\Gamma2}^2\right)^{\Psi_2}+\left(\rho_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(\rho_{\Gamma2}^2\right)^{\Psi_2}}},\dfrac{\sqrt{2}(\rho_{\Omega1}\cdot\rho_{\Omega2})}{\sqrt{\left(2-\rho_{\Omega1}^2\right)^{\Psi_1}\cdot\left(2-\rho_{\Omega2}^2\right)^{\Psi_2}+\left(\rho_{\Omega1}^2\right)^{\Psi_1}\cdot\left(\rho_{\Omega2}^2\right)^{\Psi_2}}}\right), \\
\left(\dfrac{\sqrt{2}(\sigma_{\Gamma1}\cdot\sigma_{\Gamma2})}{\sqrt{\left(2-\sigma_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(2-\sigma_{\Gamma2}^2\right)^{\Psi_2}+\left(\sigma_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(\sigma_{\Gamma2}^2\right)^{\Psi_2}}},\dfrac{\sqrt{2}(\sigma_{\Omega1}\cdot\sigma_{\Omega2})}{\sqrt{\left(2-\sigma_{\Omega1}^2\right)^{\Psi_1}\cdot\left(2-\sigma_{\Omega2}^2\right)^{\Psi_2}+\left(\sigma_{\Omega1}^2\right)^{\Psi_1}\cdot\left(\sigma_{\Omega2}^2\right)^{\Psi_2}}}\right), \\
\left(\sqrt{\dfrac{\left(1+\upsilon_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(1+\upsilon_{\Gamma2}^2\right)^{\Psi_2}-\left(1-\upsilon_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(1-\upsilon_{\Gamma2}^2\right)^{\Psi_2}}{\left(1+\upsilon_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(1+\upsilon_{\Gamma2}^2\right)^{\Psi_2}+\left(1-\upsilon_{\Gamma1}^2\right)^{\Psi_1}\cdot\left(1-\upsilon_{\Gamma2}^2\right)^{\Psi_2}}},\sqrt{\dfrac{\left(1+\upsilon_{\Omega1}^2\right)^{\Psi_1}\cdot\left(1+\upsilon_{\Omega2}^2\right)^{\Psi_2}-\left(1-\upsilon_{\Omega1}^2\right)^{\Psi_1}\cdot\left(1-\upsilon_{\Omega2}^2\right)^{\Psi_2}}{\left(1+\upsilon_{\Omega1}^2\right)^{\Psi_1}\cdot\left(1+\upsilon_{\Omega2}^2\right)^{\Psi_2}+\left(1-\upsilon_{\Omega1}^2\right)^{\Psi_1}\cdot\left(1-\upsilon_{\Omega2}^2\right)^{\Psi_2}}}\right)
\end{bmatrix}.
$$

Thus the Equation , is true for $r = 2$.

Consider that Equation , is true for $r = q$, we have,

$$
SFEZNWG\left(Q_1,Q_2,...,Q_q\right)=\begin{bmatrix}
\left(\dfrac{\sqrt{2}\prod_{r=1}^{q}(\rho_{\Gamma r})^{\Psi_r}}{\sqrt{\prod_{r=1}^{q}\left(2-\rho_{\Gamma r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(\rho_{\Gamma r}^2\right)^{\Psi_r}}},\dfrac{\sqrt{2}\prod_{r=1}^{q}(\rho_{\Omega r})^{\Psi_r}}{\sqrt{\prod_{r=1}^{q}\left(2-\rho_{\Omega r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(\rho_{\Omega r}^2\right)^{\Psi_r}}}\right), \\
\left(\dfrac{\sqrt{2}\prod_{r=1}^{q}(\sigma_{\Gamma r})^{\Psi_r}}{\sqrt{\prod_{r=1}^{q}\left(2-\sigma_{\Gamma r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(\sigma_{\Gamma r}^2\right)^{\Psi_r}}},\dfrac{\sqrt{2}\prod_{r=1}^{q}(\sigma_{\Omega r})^{\Psi_r}}{\sqrt{\prod_{r=1}^{q}\left(2-\sigma_{\Omega r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(\sigma_{\Omega r}^2\right)^{\Psi_r}}}\right), \\
\left(\dfrac{\sqrt{\prod_{r=1}^{q}\left(1+\upsilon_{\Gamma r}^2\right)^{\Psi_r}-\prod_{r=1}^{q}\left(1+\upsilon_{\Gamma r}^2\right)^{\Psi_r}}}{\sqrt{\prod_{r=1}^{q}\left(1+\upsilon_{\Gamma r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(1+\upsilon_{\Gamma r}^2\right)^{\Psi_r}}},\dfrac{\sqrt{\prod_{r=1}^{q}\left(1+\upsilon_{\Omega r}^2\right)^{\Psi_r}-\prod_{r=1}^{q}\left(1+\upsilon_{\Omega r}^2\right)^{\Psi_r}}}{\sqrt{\prod_{r=1}^{q}\left(1+\upsilon_{\Omega r}^2\right)^{\Psi_r}+\prod_{r=1}^{q}\left(1+\upsilon_{\Omega r}^2\right)^{\Psi_r}}}\right)
\end{bmatrix}
$$

The next step is to demonstrate that is true for $r = q + 1$. To this end, we have

$$SFEZNWG\left(Q_1, Q_2, ..., Q_q, Q_{q+1}\right) = \sum_{r=1}^{z} Q_{ri}^{\Psi_r} + Q_{q+1}^{\Psi_{q+1}}$$

$$= \left[\begin{array}{c} \left(\frac{\sqrt{2}\prod\limits_{r=1}^{q}\left(\rho_{\Gamma r}\right)^{\Psi_r}}{\sqrt{\prod\limits_{r=1}^{q}\left(2-\rho_{\Gamma r}^2\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(\rho_{\Gamma r}^2\right)^{\Psi_r}}}, \frac{\sqrt{2}\prod\limits_{r=1}^{q}\left(\rho_{\Omega r}\right)^{\Psi_r}}{\sqrt{\prod\limits_{r=1}^{q}\left(2-\rho_{\Omega r}^2\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(\rho_{\Omega r}^2\right)^{\Psi_r}}}\right), \\ \left(\frac{\sqrt{2}\prod\limits_{r=1}^{q}\left(\sigma_{\Gamma r}\right)^{\Psi_r}}{\sqrt{\prod\limits_{r=1}^{q}\left(2-\sigma_{\Gamma r}^2\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(\sigma_{\Gamma r}^2\right)^{\Psi_r}}}, \frac{\sqrt{2}\prod\limits_{r=1}^{q}\left(\sigma_{\Omega r}\right)^{\Psi_r}}{\sqrt{\prod\limits_{r=1}^{q}\left(2-\sigma_{\Omega r}^2\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(\sigma_{\Omega r}^2\right)^{\Psi_r}}}\right), \\ \left(\frac{\sqrt{\prod\limits_{r=1}^{q}\left(1+\upsilon_{\Gamma r}^2\right)^{\Psi_r}-\prod\limits_{r=1}^{q}\left(1-\upsilon_{\Gamma r}^2\right)^{\Psi_r}}}{\sqrt{\prod\limits_{r=1}^{q}\left(1+\upsilon_{\Gamma r}^2\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(1-\upsilon_{\Gamma r}^2\right)^{\Psi_r}}}, \frac{\sqrt{\prod\limits_{r=1}^{q}\left(1+\upsilon_{\Omega r}^2\right)^{\Psi_r}-\prod\limits_{r=1}^{q}\left(1-\upsilon_{\Omega r}^2\right)^{\Psi_r}}}{\sqrt{\prod\limits_{r=1}^{q}\left(1+\upsilon_{\Omega r}^2\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(1-\upsilon_{\Omega r}^2\right)^{\Psi_r}}}\right) \end{array}\right]$$

$$+ \left[\begin{array}{c} \left(\frac{\sqrt{2}\left(\rho_{\Gamma(q+1)}\right)^{\Psi_{q+1}}}{\sqrt{\left(2-\rho_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}+\left(\rho_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}}}, \frac{\sqrt{2}\left(\rho_{\Omega(q+1)}\right)^{\Psi_{q+1}}}{\sqrt{\left(2-\rho_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}+\left(\rho_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}}}\right), \\ \left(\frac{\sqrt{2}\left(\sigma_{\Gamma(q+1)}\right)^{\Psi_{q+1}}}{\sqrt{\left(2-\sigma_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}+\left(\sigma_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}}}, \frac{\sqrt{2}\left(\sigma_{\Omega(q+1)}\right)^{\Psi_{q+1}}}{\sqrt{\left(2-\sigma_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}+\left(\sigma_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}}}\right), \\ \left(\frac{\sqrt{\left(1+\upsilon_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}-\left(1-\upsilon_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}}}{\sqrt{\left(1+\upsilon_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}+\left(1-\upsilon_{\Gamma(q+1)}^2\right)^{\Psi_{q+1}}}}, \frac{\sqrt{\left(1+\upsilon_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}-\left(1-\upsilon_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}}}{\sqrt{\left(1+\upsilon_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}+\left(1-\upsilon_{\Omega(q+1)}^2\right)^{\Psi_{q+1}}}}\right) \end{array}\right]$$

$$= \left[\begin{array}{c} \left(\frac{\sqrt{2}\prod\limits_{r=1}^{q}\left(\rho_{\Gamma r}\right)^{\Psi_r}}{\sqrt{\prod\limits_{r=1}^{q}\left(2-\rho_{\Gamma r}^2\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(\rho_{\Gamma r}^2\right)^{\Psi_r}}}, \frac{\sqrt{2}\prod\limits_{r=1}^{q}\left(\rho_{\Omega r}\right)^{\Psi_r}}{\sqrt{\prod\limits_{r=1}^{q}\left(2-\rho_{\Omega r}^2\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(\rho_{\Omega r}^2\right)^{\Psi_r}}}\right), \\ \left(\frac{\sqrt{2}\prod\limits_{r=1}^{q}\left(\sigma_{\Gamma r}\right)^{\Psi_r}}{\sqrt{\prod\limits_{r=1}^{q}\left(2-\sigma_{\Gamma r}^2\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(\sigma_{\Gamma r}^2\right)^{\Psi_r}}}, \frac{\sqrt{2}\prod\limits_{r=1}^{q}\left(\sigma_{\Omega r}\right)^{\Psi_r}}{\sqrt{\prod\limits_{r=1}^{q}\left(2-\sigma_{\Omega r}^2\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(\sigma_{\Omega r}^2\right)^{\Psi_r}}}\right), \\ \left(\frac{\sqrt{\prod\limits_{r=1}^{q}\left(1+\upsilon_{\Gamma r}^2\right)^{\Psi_r}-\prod\limits_{r=1}^{q}\left(1-\upsilon_{\Gamma r}^2\right)^{\Psi_r}}}{\sqrt{\prod\limits_{r=1}^{q}\left(1+\upsilon_{\Gamma r}^2\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(1-\upsilon_{\Gamma r}^2\right)^{\Psi_r}}}, \frac{\sqrt{\prod\limits_{r=1}^{q}\left(1+\upsilon_{\Omega r}^2\right)^{\Psi_r}-\prod\limits_{r=1}^{q}\left(1-\upsilon_{\Omega r}^2\right)^{\Psi_r}}}{\sqrt{\prod\limits_{r=1}^{q}\left(1+\upsilon_{\Omega r}^2\right)^{\Psi_r}+\prod\limits_{r=1}^{q}\left(1-\upsilon_{\Omega r}^2\right)^{\Psi_r}}}\right) \end{array}\right].$$

thats is, when $r = q + 1$, Equation also holds.

Hence, Equation valid for any r.

By using Definition , the following characteristics of the SFEZNWG operator can be easily demonstrated.

1)Let $Q_r = \{\rho\left(\Gamma, \Omega\right)(m), \sigma\left(\Gamma, \Omega\right)(m), \upsilon\left(\Gamma, \Omega\right)(m)\} \in SFEZN$ where $r = (1, 2, 3, ..., u)$, if

$Q_1 = Q_2 = ...Q_{u-1} = Q_u = Q$, then $SFEZNWG(Q_1, Q_2, Q_3, ..., Q_u) = Q$.

2)Let $Q_r = \{\rho\left(\Gamma, \Omega\right)(m), \sigma\left(\Gamma, \Omega\right)(m), \upsilon\left(\Gamma, \Omega\right)(m)\} \in SFEZN$ where $r = (1, 2, 3, ..., u)$ and $Q^{+} = \max Q_r$

$,Q^{-} = \min Q_r$. where $Q^{-} \leq SFEZNWG(Q_1, Q_2, Q_3, ..., Q_u) \leq Q^{+}$.

3)Let $Q_r = \{\rho(\Gamma,\Omega)(m), \sigma(\Gamma,\Omega)(m), \upsilon(\Gamma,\Omega)(m)\}$ and

$Q_r^{\tilde{}} = \left\{\rho^{\tilde{}}\,(\Gamma,\Omega)(m), \sigma^{\tilde{}}\,(\Gamma,\Omega)(m), \upsilon^{\tilde{}}\,(\Gamma,\Omega)(m)\right\} \in SFEZNWG\left(Q, Q^{\tilde{}} \in M\right)$

$i.e. Q_r \leq Q_r^{\tilde{}} \forall r$.Then,$SFEZNWG(Q_1, Q_2, Q_3, ..., Q_u) \leq SFEZNWG(Q_1^{\tilde{}}, Q_2^{\tilde{}}, Q_3^{\tilde{}}, ..., Q_u^{\tilde{}})$.

Next, we suggest the SFEZNOWG (Spherical Fuzzy Einstein Z-Number Ordered Weighted Geometric) operator as follows:

4.8 Definition:

Let $Q_r = (Q = (\rho(\Gamma,\Omega)(m), \sigma(\Gamma,\Omega)(m), \upsilon(\Gamma,\Omega)(m)))\,(r = 1, 2, 3, ..., u)$.

The Einstein Ordered weighted geometric operator is characterized as

$$SFEZNOWG(Q_1, Q_2, Q_3, ..., Q_u) = \prod_{r=1}^{u} \left(Q_{\Phi(r)}\right)^{\Psi_r}$$

where $\Phi(r)$ represent the Order and $(\Phi(1), \Phi(2), ..., \Phi(u))$ is a permutation of $(1, 2, ..., u)$, subject to $Q_{\Phi(r-1)} \geq Q_{\Phi(r)} \forall r$. Also $\Psi_r\,(r \in M)$ is the weight vector with $\Psi_r \geq 0$, $\sum\limits_{r=1}^{u} \Psi_r = 1$.

4.9 Theorem:

Let $Q_r = \{\rho(\Gamma,\Omega)(m), \sigma(\Gamma,\Omega)(m), \upsilon(\Gamma,\Omega)(m)\}\,(r \in M)$.The and the $\Psi_r\,(r \in M)$ is the weight vector with $\Psi_r \geq 0$, $\sum\limits_{r=1}^{u} \Psi_r = 1$.The SFEZNOWG is a mapping $D^u \to D$ defined as:

$SFEZNOWG(Q_1, Q_2, Q_3, ..., Q_u) = \sum\limits_{r=1}^{u} \left(Q_{\Phi(r)}\right)^{\Psi_r}$

$$= \left(\begin{array}{c} \left(\dfrac{\sqrt{2} \prod\limits_{r=1}^{u} \left(\rho_{\Phi(\Gamma i)}\right)^{\Psi}}{\sqrt{\prod\limits_{r=1}^{u} \left(2-\rho_{\Phi(\Gamma i)}\right)^{\Psi} + \prod\limits_{r=1}^{u} \left(\rho_{\Phi(\Gamma i)}\right)^{\Psi}}}, \dfrac{\sqrt{2} \prod\limits_{r=1}^{u} \left(\rho_{\Phi(\Omega i)}\right)^{\Psi}}{\sqrt{\prod\limits_{r=1}^{u} \left(2-\rho_{\Phi(\Omega i)}\right)^{\Psi} + \prod\limits_{r=1}^{u} \left(\rho_{\Phi(\Omega i)}\right)^{\Psi}}} \right), \\ \left(\dfrac{\sqrt{2} \prod\limits_{r=1}^{u} \left(\sigma_{\Phi(\Gamma i)}\right)^{\Psi}}{\sqrt{\prod\limits_{r=1}^{u} \left(2-\sigma_{\Phi(\Gamma i)}\right)^{\Psi} + \prod\limits_{r=1}^{u} \left(\sigma_{\Phi(\Gamma i)}\right)^{\Psi}}}, \dfrac{\sqrt{2} \prod\limits_{r=1}^{u} \left(\sigma_{\Phi(\Omega i)}\right)^{\Psi}}{\sqrt{\prod\limits_{r=1}^{u} \left(2-\sigma_{\Phi(\Omega i)}\right)^{\Psi} + \prod\limits_{r=1}^{u} \left(\sigma_{\Phi(\Omega i)}\right)^{\Psi}}} \right), \\ \left(\dfrac{\sqrt{\prod\limits_{r=1}^{u} \left(1+\upsilon_{\Phi(\Gamma i)}^2\right)^{\Psi} - \prod\limits_{r=1}^{u} \left(1-\upsilon_{\Phi(\Gamma i)}^2\right)^{\Psi}}}{\sqrt{\prod\limits_{r=1}^{u} \left(1+\upsilon_{\Phi(\Gamma i)}^2\right)^{\Psi} + \prod\limits_{r=1}^{u} \left(1-\upsilon_{\Phi(\Gamma i)}^2\right)^{\Psi}}}, \dfrac{\sqrt{\prod\limits_{r=1}^{u} \left(1+\upsilon_{\Phi(\Omega i)}^2\right)^{\Psi} - \prod\limits_{r=1}^{u} \left(1-\upsilon_{\Phi(\Omega i)}^2\right)^{\Psi}}}{\sqrt{\prod\limits_{r=1}^{u} \left(1+\upsilon_{\Phi(\Omega i)}^2\right)^{\Psi} + \prod\limits_{r=1}^{u} \left(1-\upsilon_{\Phi(\Omega i)}^2\right)^{\Psi}}} \right) \end{array} \right).$$

where $\Phi(r)$ is denoted for Ordered and $(\Phi(1), \Phi(2), \Phi(3), ..., \Phi(u))$ is a permutation of $(1, 2, 3, ..., u)$, subject to $Q_{\Phi(r-1)} \geq Q_{\Phi(r)}$ for all r.

Proof: Since the proof of this theorem is the same as that of Theorem , it is not included here.

By using Definition , the following characteristics of the SFEZNOWG operator can be easily demonstrated.

1)Let $Q_r = \{\rho(\Gamma,\Omega)(m), \sigma(\Gamma,\Omega)(m), \upsilon(\Gamma,\Omega)(m)\} \in SFEZN$ where $r = (1, 2, ..., u)$, if

$Q_1 = Q_2 = ...Q_{u-1} = Q_u = Q$, then $SFEZNOWG(Q_1, Q_2, ..., Q_u) = Q$.

2)Let $Q_r = \{\rho(\Gamma,\Omega)(m), \sigma(\Gamma,\Omega)(m), \upsilon(\Gamma,\Omega)(m)\} \in SFEZN$ where $r = (1,2,...,u)$ and $Q^{+} = \max Q_r$

,$Q^{-} = \min Q_r$. where $Q^{-} \leq SFEZNOWG(Q_1, Q_2, ..., Q_u) \leq Q^{+}$.

3)Let $Q_r = \{\rho(\Gamma,\Omega)(m), \sigma(\Gamma,\Omega)(m), \upsilon(\Gamma,\Omega)(m)\}$ and

$Q_r^{\tilde{}} = \left\{\rho^{\tilde{}}(\Gamma,\Omega)(m), \sigma^{\tilde{}}(\Gamma,\Omega)(m), \upsilon^{\tilde{}}(\Gamma,\Omega)(m)\right\} \in SFEZNOWG\left(Q, Q^{\tilde{}} \in M\right)$

$i.e. Q_r \leq Q_r^{\tilde{}} \forall r$.Then,$SFEZNOWG(Q_1, Q_2, ..., Q_u) \leq SFEZNOWG(Q_1^{\tilde{}}, Q_2^{\tilde{}}, ..., Q_u^{\tilde{}})$.

5. Algorithm for Decision-Making

A matrix can be used to display the issues associated with multi-attribute decision-making. The columns & rows of a matrix are defined by attributes, and the alternatives are indicated.The matrix $D_{h\times j}$, Take a look at a collection of h alternatives $\{T_1, T_2, T_3, ..., T_h\}$and j criteria attributes $\{K_1, K_2, K_3, ..., K_j\}$ .Weights for attributes are described as $W = \{w_1, w_2, w_3, ..., w_j\}$ as $w_g \in [0,1], \sum_{g=1}^{m} w_g = 1$.Consider the matrix $r = (Q_{mn})_{h\times j} = \{\rho_{mn}(\Gamma,\Omega), \sigma_{mn}(\Gamma,\Omega), \upsilon_{mn}(\Gamma,\Omega)\}$ access data from Spherical Fuzzy Z-Numbers. In order to create an algorithm for solving spherical fuzzy Z number settings decision-making problems.

$$D_{h\times j} = \begin{array}{c} T_1 \\ \\ T_2 \\ \\ \vdots \\ T_h \end{array} \left[\begin{array}{cccc} K_1 & K_2 & & K_J \\ \left\langle \begin{matrix} (\rho_{\Gamma_{11}}, \rho_{\Omega_{11}}), \\ (\sigma_{\Gamma_{11}}, \sigma_{\Omega_{11}}), \\ (\upsilon_{\Gamma_{11}}, \upsilon_{\Omega_{11}}) \end{matrix} \right\rangle & \left\langle \begin{matrix} (\rho_{\Gamma_{12}}, \rho_{\Omega_{12}}), \\ (\sigma_{\Gamma_{12}}, \sigma_{\Omega_{12}}), \\ (\upsilon_{\Gamma_{12}}, \upsilon_{\Omega_{12}}) \end{matrix} \right\rangle & \cdots & \left\langle \begin{matrix} (\rho_{\Gamma_{1j}}, \rho_{\Omega_{1j}}), \\ (\sigma_{\Gamma_{1j}}, \sigma_{\Omega_{1j}}), \\ (\upsilon_{\Gamma_{1j}}, \upsilon_{\Omega_{1j}}) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} (\rho_{\Gamma_{21}}, \rho_{\Omega_{21}}), \\ (\sigma_{\Gamma_{21}}, \sigma_{\Omega_{21}}), \\ (\upsilon_{\Gamma_{21}}, \upsilon_{\Omega_{21}}) \end{matrix} \right\rangle & \left\langle \begin{matrix} (\rho_{\Gamma_{22}}, \rho_{\Omega_{22}}), \\ (\sigma_{\Gamma_{22}}, \sigma_{\Omega_{22}}), \\ (\upsilon_{\Gamma_{22}}, \upsilon_{\Omega_{22}}) \end{matrix} \right\rangle & \cdots & \left\langle \begin{matrix} (\rho_{\Gamma_{2j}}, \rho_{\Omega_{2j}}), \\ (\sigma_{\Gamma_{2j}}, \sigma_{\Omega_{2j}}), \\ (\upsilon_{\Gamma_{2j}}, \upsilon_{\Omega_{2j}}) \end{matrix} \right\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \left\langle \begin{matrix} (\rho_{\Gamma_{h1}}, \rho_{\Omega_{h1}}), \\ (\sigma_{\Gamma_{h1}}, \sigma_{\Omega_{h1}}), \\ (\upsilon_{\Gamma_{h1}}, \upsilon_{\Omega_{h1}}) \end{matrix} \right\rangle & \left\langle \begin{matrix} (\rho_{\Gamma_{h2}}, \rho_{\Omega_{h2}}), \\ (\sigma_{\Gamma_{h2}}, \sigma_{\Omega_{h2}}), \\ (\upsilon_{\Gamma_{h2}}, \upsilon_{\Omega_{h2}}) \end{matrix} \right\rangle & \cdots & \left\langle \begin{matrix} (\rho_{\Gamma_{hj}}, \rho_{\Omega_{hj}}), \\ (\sigma_{\Gamma_{hj}}, \sigma_{\Omega_{hj}}), \\ (\upsilon_{\Gamma_{hj}}, \upsilon_{\Omega_{hj}}) \end{matrix} \right\rangle \end{array} \right]$$

5.1 Method of aggregation using Einstein norms:

We put out a technique for SFEZNAO's to handle MADM issues in relation to SFZN situations. Following are the algorithm's fundamental steps:

Step 1 SFZN in the DM are as shown in Table

Step 2 In order to aggregate the spherical fuzzy Z-Numbers information, this step involves applying aggregation operators to the decision matrix.Attributes weights are $w = (0.26, 0.41, 0.33)$.

Step 2(i) Application of SFEZNWA Aggregation operator to gather spherical fuzzy Z-Numbers data.

Step 2(ii) Application of SFEZNOWA Aggregation operator to gather spherical fuzzy Z-Numbers data.

Step 2(iii) Application of SFEZNWG Aggregation operator to gather spherical fuzzy Z-Numbers data.

Step 2(iv) Application of SFEZNOWG Aggregation operator to gather spherical fuzzy Z-Numbers data.

Step 3 Find the scores of the combined spherical fuzzy Z-Numbers and sort by highest value.

Step 4 Determine which option has the highest score value, and select that.

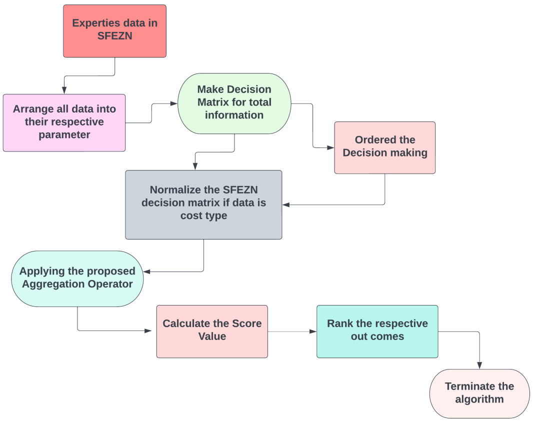

Fig. 1. Algorithm

5.2 Numerical Illustration:

With the help of a numerical model, the discussed technique has been demonstrated; the specifics are given below.

A dam is an obstruction in the path of moving water that restricts, directs, or slows the flow, frequently resulting in the creation of a lake, reservoir, or impoundments. Due to the potential catastrophic effects on the civilian population and the environment, dams are classified as "installations containing dangerous forces" under international humanitarian law. Although relatively infrequent, dam failures can result in significant damage and fatalities.When the Banqiao Reservoir Dam and other dams in China's Henan Province broke in 1975, more people perished than ever before in a dam failure. More people were killed in China than in any other dam failure in history. 11 million people lost their homes, and the disaster is estimated to have killed 171,000 people.

In addition, many dam accidents are a result of other natural calamities like earthquakes, landslides, severe storms, or rapid snowmelt. Sabotage, structural harm, and equipment malfunction are further factorsCommon causes of dam failure include:

- Sliding of a mountain into the reservoir.
- Poor maintenance, especially of outlet pipes (Lawn Lake Dam).
- Extreme inflow (Shakidor Dam).
- Earthquakes.
- Climate-driven landscape instability.

Emergency intervention techniques can be applied to try to prevent a dam from completely failing. Examples include using sandbags to increase freeboard and prevent overtopping, riprap to stop erosion of the dam structure, and a geotextile filter fabric to stop piping.In an emergency response to a dam crisis, there are five key steps: (1) Planning, (2) Evaluation, (3) Monitoring, (4) Reaction, and (5) Post-Action Documentation and Follow-Up.

For this we Consider that the matrix $D_{h\times j}$, consider a collection of h alternatives $\{T_1, T_2, T_3, ..., T_h\}$and j criteria attributes $\{K_1, K_2, K_3, ..., K_j\}$ .Weights for attributes are described as $w = (0.26, 0.41, 0.33)$.The best option is then presented to us, one that is the greatest benefit dam.*Masonry Dam* (T_1)any dam whose primary building materials are mortared stone, brick, or concrete blocks. It is incorrect to call a dam a masonry dam if its only masonry facing is present. A murky dam is made of uncoursed or irregularly shaped stones. *Overflow Dam* (T_2)a dam intended for overtopping construction.Hydropower Dam(T_3)a dam that powers an electricity-generating turbine by using the difference in water levels between the reservoir pool elevation and the tailwater elevation. Consider a committee of decision-makers to conduct the assessment and choose the K_1, K_2, K_3, K_4 attributes that best fits the situation. The following four attributes are used by the decision-maker is:

- Availability of Materials.
- Spillway Size and Location.
- Earthquake Zone.
- Height of Dam.

As can be seen in Table 1, three alternatives can be rated as suitable for T_1, T_2, T_3 under each of the three attributes K_1, K_2, K_3.

By using SFEZNWA and SFEZNOWA Operator:

Step 1 The information given by the expert in the SFEZNs form is represented in the Table 1.

Table 1: Decision matrix by the expert

$$\begin{array}{cccc}
 & K_1 & K_2 & K_3 \\
T_1 & \left\langle \begin{array}{c} (0.2,0.3), \\ (0.5,0.3), \\ (0.3,0.1) \end{array} \right\rangle & \left\langle \begin{array}{c} (0.4,0.5), \\ (0.2,0.3), \\ {}'(0.5,0.5) \end{array} \right\rangle & \left\langle \begin{array}{c} (0.2,0.2), \\ (0.6,0.4), \\ (0.5,0.4) \end{array} \right\rangle \\
T_2 & \left\langle \begin{array}{c} (0.2,0.4), \\ (0.3,0.4), \\ (0.1,0.2) \end{array} \right\rangle & \left\langle \begin{array}{c} (0.6,0.2), \\ (0.5,0.3), \\ (0.2,0.6) \end{array} \right\rangle & \left\langle \begin{array}{c} (0.5,0.3), \\ (0.4,0.1), \\ (0.2,0.6) \end{array} \right\rangle \\
T_3 & \left\langle \begin{array}{c} (0.1,0.2), \\ (0.2,0.3), \\ (0.5,0.2) \end{array} \right\rangle & \left\langle \begin{array}{c} (0.7,0.2), \\ (0.4,0.5), \\ (0.1,0.2) \end{array} \right\rangle & \left\langle \begin{array}{c} (0.4,0.3), \\ (0.5,0.5), \\ (0.2,0.7) \end{array} \right\rangle
\end{array}$$

Step 2 Normalization is unnecessary due to the benefit-type nature of the criterion.

Step 3 Integrate the attributes for each alternative using SFEZNWA operator, represented in Table 2.

Table 2: Total Evaluation of SFEZNWA Operator

	K_j
T_1	$\langle(0.2992, 0.3747), (0.3715, 0.3302), (0.4398, 0.3109)\rangle$
T_2	$\langle(0.4941, 0.2965), (0.4082, 0.2268), (0.1671, 0.4600)\rangle$
T_3	$\langle(0.5185, 0.2377), (0.3623, 0.4398), (0.1933, 0.3110)\rangle$

Step 4 Review each SFEZN's score value and then re-order the SFEZN against each attribute as represented in Table 3.

Table 3: Re-ordered Decision matrix

$$K_1 \qquad K_2 \qquad K_3$$

$$\begin{array}{cccc}
T_1 & \left\langle\begin{array}{c}(0.4,0.5),\\(0.2,0.3),\\(0.5,0.5)\end{array}\right\rangle & \left\langle\begin{array}{c}(0.2,0.3),\\(0.5,0.3),\\(0.3,0.1)\end{array}\right\rangle & \left\langle\begin{array}{c}(0.2,0.2),\\(0.6,0.4),\\(0.5,0.4)\end{array}\right\rangle \\
T_2 & \left\langle\begin{array}{c}(0.5,0.3),\\(0.4,0.1),\\(0.2,0.6)\end{array}\right\rangle & \left\langle\begin{array}{c}(0.2,0.4),\\(0.3,0.4),\\(0.1,0.2)\end{array}\right\rangle & \left\langle\begin{array}{c}(0.6,0.2),\\(0.5,0.3),\\(0.2,0.6)\end{array}\right\rangle \\
T_3 & \left\langle\begin{array}{c}(0.7,0.2),\\(0.4,0.5),\\(0.1,0.2)\end{array}\right\rangle & \left\langle\begin{array}{c}(0.1,0.2),\\(0.2,0.3),\\(0.5,0.2)\end{array}\right\rangle & \left\langle\begin{array}{c}(0.4,0.3),\\(0.5,0.5),\\(0.2,0.7)\end{array}\right\rangle
\end{array}$$

Step 5 Integrate the attributes for each alternative using SFEZNOWA operator, represented in Table 4

Table4: Total Evaluation of SFEZNOWA Operator

	K_j
T_1	$\langle(0.2673,0.3407),(0.4249,0.3302),(0.4077,0.2442)\rangle$
T_2	$\langle(0.4519,0.3201),(0.3841,0.2559),(0.1507,0.3912)\rangle$
T_3	$\langle(0.4407,0.2377),(0.3271,0.4077),(0.2470,0.3110)\rangle$

Step 6 Determine the score values by using SFEZNWA and SFEZNOWA operator in table 5 and table 6:

Table 5: Score Value forSFEZNWA

	K_j
T_1	$\langle 0.6175\rangle$
T_2	$\langle 0.6590\rangle$
T_3	$\langle 0.6345\rangle$

Table 6: Score Value for SFEZNOWA

	K_j
$Y(T_1)$	$\langle 0.6170\rangle$
$Y(T_2)$	$\langle 0.6624\rangle$
$Y(T_3)$	$\langle 0.6315\rangle$

Step 7 Ranking each valueable option in descending order, select the option that is most preferable.By SFEZNWA operator:

$$Y(T_2) \succ Y(T_3) \succ Y(T_1).$$

By using SFEZNOWA operator :

$$Y(T_2) \succ Y(T_3) \succ Y(T_1).$$

We conclude that option T_2 is the best option as a result.

By using SFEZNWG and SFEZNOWG Operator

Step 1 The information given by the expert in the SFEZNs form is represented in the table 1.

Step 2 Normalization is unnecessary due to the benefit-type nature of the criterion.

Step 3 Integrate the attributes for each alternative using SFEZNWG operator in table 7.

Table 7: Total Evaluation of SFEZNWG Operator

	K_j
T_1	$\langle (0.2670, 0.3265), (0.3715, 0.3302), (0.4575, 0.3990) \rangle$
T_2	$\langle (0.4316, 0.2747), (0.4082, 0.2268), 0.1794, 0.5305 \rangle$
T_3	$\langle (0.3643, 0.2288), (0.3623, 0.4398), (0.2889, 0.4482) \rangle$

Step 4 Reorder the SFEZN in relation to each represented attribute after evaluating the score value of each SFEZN as shown in table 8.

Table 8: Re-ordered decision matrix

$$\begin{array}{cc} & \begin{array}{ccc} \quad K_1 \quad & \quad K_2 \quad & \quad K_3 \quad \end{array} \\ \begin{array}{c} T_1 \\ \\ \\ T_2 \\ \\ \\ T_3 \\ \\ \end{array} & \left[\begin{array}{ccc} \left\langle \begin{matrix} (0.4, 0.5), \\ (0.2, 0.3), \\ (0.5, 0.5) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.2, 0.3), \\ (0.5, 0.3), \\ (0.3, 0.1) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.2, 0.2), \\ (0.6, 0.4), \\ (0.5, 0.4) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} (0.5, 0.3), \\ (0.4, 0.1), \\ (0.2, 0.6) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.2, 0.4), \\ (0.3, 0.4), \\ (0.1, 0.2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.6, 0.2), \\ (0.5, 0.3), \\ (0.2, 0.6) \end{matrix} \right\rangle \\ \left\langle \begin{matrix} (0.7, 0.2), \\ (0.4, 0.5), \\ (0.1, 0.2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.1, 0.2), \\ (0.2, 0.3), \\ (0.5, 0.2) \end{matrix} \right\rangle & \left\langle \begin{matrix} (0.4, 0.3), \\ (0.5, 0.5), \\ (0.2, 0.7) \end{matrix} \right\rangle \end{array} \right] \end{array}$$

Step 5 Integrate the attributes for each alternative using SFEZNOWA operator, represented in Table 9.

Table 9:Total evaluation of SFEZNOWG Operator

	K_j
T_1	$\langle (0.2403, 0.3017), (0.4249, 0.3302), (0.4307, 0.3509) \rangle$
T_2	$\langle (0.3715, 0.2964), (0.3841, 0.2559), (0.1664, 0.4840) \rangle$
T_3	$\langle (0.2707, 0.2288), (0.3271, 0.4077), (0.3463, 0.4482) \rangle$

Step 6 Determine the score values by using SFEZNWG and SFEZNOWG operator in table 10 and table 11:

Table 10: Score Value for SFEZNWG

	K_j
T_1	$\langle 0.5939 \rangle$
T_2	$\langle 0.6435 \rangle$
T_3	$\langle 0.5981 \rangle$

Table 11: Score Value for SFEZNOWG

	K_j
$Y(T_1)$	$\langle 0.5936 \rangle$
$Y(T_2)$	$\langle 0.6437 \rangle$
$Y(T_3)$	$\langle 0.5911 \rangle$

Step 7 Ranking each valueable option in descending order, select the option that is most preferable.By SFEZNWG operator:

$$Y(T_2) \succ Y(T_3) \succ Y(T_1).$$

By using SFEZNOWG operator :

$$Y\left(T_2\right) \succ Y\left(T_1\right) \succ Y\left(T_3\right).$$

We conclude that option T_2 is the best option as a result.

6. Relative Comparison

A comparison of the effectiveness of the suggested algorithms with some of the current spherical fuzzy Einstein Z-Number measures has been made. The best option is the same in all approaches, despite slightly different ranking orders table 12 and in Figure 2 provide the rankings and graphical depictions of each operator:

Table 12: Relative Comparison

Operator	Ranking
SFEZNWA	$T_2 \succ T_3 \succ T_1$
SFEZNOWA	$T_2 \succ T_3 \succ T_1$
SFEZNWG	$T_2 \succ T_3 \succ T_1$
SFEZNOWG	$T_2 \succ T_1 \succ T_3$

The significance of the combined sets of spherical fuzzy Einstein Z-Numbers is simply taken into account by the weighted SFEZNs averaging operator. Only the ranking order of the combined spherical fuzzy einstein sets and the significance of each position are taken into account by the SFEZNOWA operator. The grafical representation is given in Figure 2.

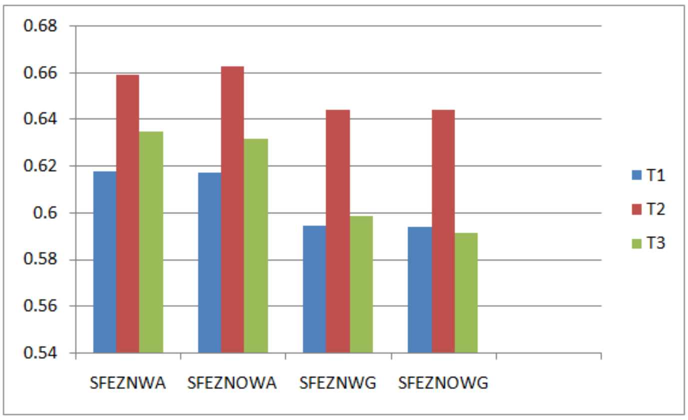

Fig. 2. Graphical representation of Operators

Since the decision-maker can select the characteristics and operators based on their particular needs and the current situation, the suggested approach is more efficient.

7. Conclusion

To assemble the dam, generalized SFEZNWA, WG, OWA, and OWG have been presented in this manuscript. These generalized Einstein aggregation operators' formal definitions and characteristics have been offered and investigated, respectively. Their precise expressions are determined by the SFEZNs' operational laws, which were developed using Einstein's t-norm and t-conorm. A fresh approach to resolving the MADM problem has been suggested based on the particular expressions. This applies the weight vector to the suggested SFEZNAOs to identify the best option based on the provided three alternatives. The proposed SFEZNAO are subjected to the weight vector, and the results of the decision-making processes are displayed in Figure 2. Given the alternatives, it is clear that T_2 is the best option.

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