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Extension of Interaction Geometric Aggregation Operator for Material Selection Using Interval-Valued Intuitionistic Fuzzy Hypersoft Set

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ABSTRACT

A recently emerged area of research, named the intuitionistic fuzzy hypersoft set (IFHSS), attempts to describe the internal limitations of intuitionistic fuzzy soft sets on multiparameter functions. A computation of such a type connects a power set of the universe with a tuple of sub-parameters. The strategy shows the allocation of attributes to their respective sub-attribute values in distinct groupings. The above features make it a unique methodical tool for handling obstacles of hesitation. Aggregation operators have an important role in the assessment of both types of potential and in identifying problems from their assessment. This research extends the use of the interaction aggregation operator to the interval-valued intuitionistic fuzzy hypersoft set (IVIFHSS), which is an entirely new structure generated through the interval-valued intuitionistic fuzzy soft set (IVIFSS). The IVIFHSS significantly condenses information that is inaccurate and imprecise compared to the frequently utilized IFSS and IVIFSS. Fuzzy reasoning is recognized as the prevalent strategy for improving imperfect data in decision-making processes. The core objective of the research is to develop operational rules for interval-valued intuitionistic fuzzy hypersoft numbers (IVIFHSNs), which promote interactions. This research is designed to broaden the utilization of the interaction geometric aggregation operator in the framework of IVIFHSS. In particular, we propose a novel operator known as the Interval-Valued Intuitionistic Fuzzy Hypersoft Interactive Weighted Geometric (IVIFHSIWG) operator. The aggregation operator indicates industry professionals' support for the implementation of a robust MCGDM material selection technique in order to address this need. The practical application of the intended MCGDM technique has been introduced for selecting materials (MS) for cryogenic storage containers. The findings indicate that the anticipated model is more operational and stable in demonstrating uncertain facts based on IVIFHSS.

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1. Introduction

The Multi-Criteria Group Decision Making (MCGDM) method has been considered the best strategy for managing ordinary choices due to its ability to consider many options, criteria, and frameworks. Most verdicts are reserved when the objectives and boundaries are ordinarily unstipulated or indistinct in actual surroundings. Zadeh [1] originated the theory of a fuzzy set (FS) to incredulous these nebulous and tentative details. It is a fundamental theory to covenant with inconsequential and doubtful statistics in the decision-making (DM) procedure. The prevailing FS fails to effectively engage with nations in which experts frequently consider membership (MD) through the DM process. Turksen [2] developed an interval-valued fuzzy set (IVFS) with fundamental operations. The present structures of FS and IVFS are insufficient in providing empirical evidence for the concept of an alternative non-membership degree (NMD). Atanassov [3] overcame such constraints and introduced the concept of intuitionistic fuzzy sets (IFS). Wang and Liu [4] presented numerous operations, such as the Einstein product, Einstein, and others, and the AOs of the IFS. Garg and Rani [5] used provided AOs to solve the MULTIMOORA technique under IFS information. Ejegwa and Agbetayo [6] investigated different approaches for similarity measures (SM) and distance measures in the framework of IFS. They used the proposed measures to deal with obstacles faced in DM. Atanassov [7] extended upon the idea of IFS with the term of an interval-valued IFS (IVIFS), which includes fundamental operations and associated specifications. Gurmani et al. [8] developed the TOPSIS method for interval-valued T-spherical fuzzy sets and used it to select the most suitable construction company. Xu and Gou [9] established numerous DM approaches under the IVIFS setup and used their methods for several real-life difficulties. Ze-Shui [10] designed a weighted geometric aggregation operator for IVIFS. Zhang [11] presented the Bonferroni mean geometric aggregation operator in the context of IVIFS structure and put forth the MAGDM technique. Park et al. [12] developed a Bonferroni mean geometric aggregation operator for IVIFS in order to tackle MAGDM challenges. Gupta et al. [13] proposed an appropriate structure for managing the impact of expert weight. Garg and Kumar [14] protracted the basic features of AOs in the context of the linguistic IVIFS.

Although parametric chemistry has multiple uses, its potential is limited by the shortcomings of the theories mentioned above. To deal with vagueness and obscurity, Molodtsov [15] introduced the idea of soft sets (SS) and clarified specific operations with their attributes. Maji et al. [16] extended the idea of SS and provided multiple illustrations to illustrate fundamental SS processes. The idea of fuzzy soft sets (FSS) with desirable features was introduced by Maji et al. [17], who integrated the popular theories of FS and SS. Maji et al. [18] enhanced the notion of intuitionistic fuzzy soft sets (IFSS) by introducing multiple fundamental operations. Arora and Garg [19] proposed the AOs and Deliberate DM approach, which was developed on the operators they had designed. The IVIFSS was introduced by Jiang et al. [20], and its fundamental attributes were examined and analyzed. The TOPSIS approach based on the correlation coefficient (CC) was developed by Zulqarnain et al. [21] in order to address MADM difficulties in the context of the IVIFSS. Zulqarnain and Saeed [22] proposed the concept of an interval-valued fuzzy soft matrix and applied it in DM. Ma et al. [23] protracted a novel DM methodology for IVIFSS and used their proven method to solve DM obstacles. The theory of hypersoft sets (HSS) was predicted by Smarandache [24], and it is characterized by the parametric function f that is unique to the cartesian product with n attributes. Samarandche HSS is the most accurate model related to SS and other fixed ideas since it captures the many qualities of the considered factors. Multiple HSS prospects exist, each with a unique DM strategy in the works. The DM techniques for IFHSS were developed by Rahman et al. [25] and based on SM. Zulqarnain et al. [26] prolonged the DM method for IFHSS based on their established AOs. Debnath [27] spread the interval-valued IFHSS (IVIFHSS) theory with fundamental operations and introduced an MCDM model to handle DM hurdles. Zulqarnain et al. [28] anticipated the AOs for IVIFHSS and extended the innovative MCGDM method with its application in material selection.

The IVIFHSS is a reasonable amalgamation structure of IVFS; IVFSS and IVIFSS are leading systematic tools for assigning unknown and controlled facts. It has been acknowledged that AOs are authoritative in DM, so mutually calculated specifics from contrasting bases can be composed in a distinct evaluation. To the unequaled of our deliberation, interaction AOs with hybridization of an HSS and IVIFS have no existence in the works. Static, prevailing AOs for IVIFHSS cannot proficiently contract the undefined and inaccurate facts through DM progression. Also, the approach claims that the complete MD (NMD) expressed as a series of intervals is an NMD (MD) that determines itself. Thus, anxieties are not beneficial, and the strategies are validated. This indicates that no apparent preference for alternatives is demonstrated.

Therefore, it is a fascinating topic to discuss how to combine these IVIFHSNs across interaction AOs. The IVIFHSS interaction AO, including the IVIFHSIWG operator, will be presented. Common amalgam expansions of FS are similar to the settled interaction AO. The models mentioned above have concluded that the NMD (MD) interval values shed light on the MD (NMD) as a whole. Therefore, the importance of these AOs is uncertain, and no supplementary data for replacements are provided. Hence, integrating these IVIFHSNs over interaction AOs is an exciting issue. The techniques employed in [21, 23] are insufficient to assess the facts, commenting on established theory and evident consequences.

1.1. Motivation and Drawbacks of Existing Operators

IVIFHSS is a significant scientific approach to coping with ambiguity, coherence, and incomplete information since it is a hybrid conceptual structure of HSS, IVIFS, and IVIFSS. Cooperative assessment data from several sources can be written into specialized imposts with the use of AOs, which have been found to be crucial in DM. We were unable to find any examples in the literature of the usage of the geometric interaction AO of HSS hybridization with IVIFS. Yet, only some of the stated above ideas are acceptable for IVIFHSN extractions, nor can they be purposely connected with MD and NMD. The effect of various degrees of MD or NMD on the appropriate geometric AO does not upset the overall procedure. NMD(MD) functional level is also assumed to be determined automatically by the framework. So, if we let these copies through, we won't learn which alternatives should be prioritized. Thus, integrating these IVIFHSNs into the interaction is a stimulating problem. We will endorse the IVIFHSIWG operator to report these concerns. Since the general IVFS, IVIFS, and IVIFSS are exclusions to IVIFHSS, introduced interactive AO has more talented related to AOs with prevalent hybrid assemblies of IVFS. Therefore, currently used models produce results that are inconsistent and fail to rank preferences among available options properly. Integration of these IVIFHSNs via interaction is thus an intriguing topic. To evaluate data in resonant intelligence and get better ideas and more precise outcomes, you need more than what is provided in [27, 28]. Therefore, we declare that the newly invented IVIFHSIWG operator is a modernized class approach that intrigues detectives to crack confusing and insufficient specifications.

1.2. Significant Contribution

We design an approach for choosing MS using IVIFHSS data to alleviate these limitations. Researchers are attracted to an improved structuring strategy in the expectation of revealing absent and/or limited data that can be used to fill these deficiencies. IVIFHSS performs an influential role in the DM evaluating the exploration consequences by assembling rich foundations into a distinct value. It's an innovative hybrid structure that assists the DM mechanism in dealing with problematic challenges. So, we will choose a geometric interaction AO based on non-regular facts to kick off the current IVIFHSS inquiry. The following are the primary goals of the current study that were originally established:

The integration of numerous aspects of the matter, including the sub-attributes of indicators examined in the DM mechanism, can be achieved by the implementation of IVIFHSS. In order to preserve the advantages associated with this incorporation, we have expanded the IVIFHSIWG operator of IVIFHSS.

The IVIFHSIWG operator used in the framework of IVIFHSS is renowned as an attractive assessing computational operator. In particular circumstances, the primary component of AOs exhibits a lack of responsiveness towards accurately indicating the discovery within the DM system. In order to overcome these challenging obstacles, it is essential to modify existing AOs. We construct legal structures for the operational requirements governing IVIFHSNs.

The IVIFHSIWG operator has been introduced, highlighting its significant qualities and established operational laws governing its interaction.

Develop an innovative approach based on an assumed operator schedule to explain the MCGDM issue within the IVIFHSS structure.

The implementation of MS, or management science, is a prominent characteristic within engineering sectors due to its ability to address the specific conditions associated with many components comprehensively. Master of Science (MS) is a rigorous yet pivotal phase in professional development. The company's ability to do tasks with skill, effectiveness, and uniqueness will be negatively impacted as a result of the lack of available supplies.

This paper presents a comparison of the unique MCGDM approach and previous approaches, with the aim of evaluating their applicability and efficiency.

This article's layout is predicated on the following assumptions: Some basic ideas that underpin our development of the advanced study are covered in the second section. In Section 3, several novel algebraic operations for IVIFHSS with interaction are proposed. In the same part, the IVIFHSIWG operator will also be introduced along with its basic features. The MCGDM approach to the anticipated interaction AO is shown in Section 4. The part in question examines a numerical example of MS in engineering to support the practicality of traditional methods. A quick comparison study is also carried out to validate the viability of the approach decided in Section 5.

2. Preliminaries

A few basic definitions are included in this section to help structure the work that follows.

2.1 Definition [7]

Let U be a universe of discourse, and A be any subset of U. Then, the IVIFS A over U is defined as:

$$A = \left\{\left(x, \left(\left[\kappa_A^l(t), \kappa_A^u(t)\right], \left[\delta_A^l(t), \delta_A^u(t)\right]\right)\right) \middle| t \in U\right\}$$

Where, $\left[\kappa_A^l(t), \kappa_A^u(t)\right]$ and $\left[\delta_A^l(t), \delta_A^u(t)\right]$ represent the MD and NMD intervals, respectively. Also, $\kappa_A^l(t), \kappa_A^u(t), \delta_A^l(t), \delta_A^u(t) \in [0,1]$And satisfied the subsequent condition $0 \leq \kappa_A^u(t) + \delta_A^u(t) \leq 1$.

2.2 Definition [20]

Let $\mathbb{N}$ be a collection of attributes, and U be a discourse universe. A pair $(\Omega, \mathbb{N})$ is then considered to be an IVIFSS over U. Its mapping is articulable as

$$\Omega: \mathbb{N} \to IK^U$$

where the set of interval-valued intuitionistic fuzzy subsets of the discourse U universe is represented by IK^U.

$$(\Omega, \mathbb{N}) = \left\{x, \left(\left[\kappa_A^l(t), \kappa_A^u(t)\right], \left[\delta_A^l(t), \delta_A^u(t)\right]\right) \middle| t \in A\right\}$$

Where, $\left[\kappa_A^l(t), \kappa_A^u(t)\right], \left[\delta_A^l(t), \delta_A^u(t)\right]$ represents the MD and NMD intervals, respectively. Also, $\kappa_A^l(t), \kappa_A^u(t),\ \delta_A^l(t), \delta_A^u(t) \in [0,1]$And satisfied the subsequent condition $0 \leq \kappa_A^u(t) + \delta_A^u(t) \leq 1$ and $A \subset \mathbb{N}$.

2.3 Definition [24]

Let U be a universe of discourse and $\mathcal{P}(U)$ be a power set of U and $t = \{t_1, t_2, t_3, \ldots, t_n\}$,(n ≥ 1) and T_i represented the set of attributes and their corresponding sub-attributes, such as $T_i \cap T_j = \phi$, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1,2,3 \ldots n\}$. Assume $T_1 \times T_2 \times T_3 \times \ldots \times T_n = \ddot{A} = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$ is a collection of sub-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and $\alpha, \beta, \gamma \in$ N. Then the pair ($\mathcal{F}$, $T_1 \times T_2 \times T_3 \times \ldots \times T_n = (\Omega, \ddot{A})$ is known as IFHSS and is defined as follows:

$\Omega: T_1 \times T_2 \times T_3 \times \ldots \times T_n = \ddot{A} \rightarrow IFS^U$.

It is also defined as

$(\Omega, \ddot{A}) = \left\{\left(\check{d}, \Omega_{\ddot{A}}(\check{d})\right) : \check{d} \in \ddot{A},\ \Omega_{\ddot{A}}(\check{d}) \in IFS^U \in [0,1]\right\}$, where $\Omega_{\ddot{A}}(\check{d}) = \left\{\langle \zeta, \kappa_{\Omega(\check{d})}(\zeta), \delta_{\Omega(\check{d})}(\zeta)\rangle : \zeta \in U\right\}$, where $\kappa_{\Omega(\check{d})}(\zeta)$ and $\delta_{\Omega(\check{d})}(\zeta)$ represents the MD and NMD, respectively, such as $\kappa_{\Omega(\check{d})}(\zeta), \delta_{\Omega(\check{d})}(\zeta) \in [0,1]$, and $0 \leq \kappa_{\Omega(\check{d})}(\zeta) + \delta_{\Omega(\check{d})}(\zeta) \leq 1$.

2.4 Definition [27]

Let U be a universe of discourse and $\mathcal{P}(U)$ be a power set of U and $t = \{t_1, t_2, t_3, \ldots, t_n\}$,(n ≥ 1) and T_i represented the set of attributes and their corresponding sub-attributes, such as $T_i \cap T_j = \phi$, where $i \neq j$ for each $n \geq 1$ and $i, j \in \{1,2,3 \ldots n\}$. Assume $T_1 \times T_2 \times T_3 \times \ldots \times T_n = \ddot{A} = \{d_{1h} \times d_{2k} \times \cdots \times d_{nl}\}$ is a collection of sub-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and $\alpha, \beta, \gamma \in$ N. Then the pair ($\mathcal{F}$, $T_1 \times T_2 \times T_3 \times \ldots \times T_n = (\Omega, \ddot{A})$ is known as IVIFHSS and is defined as follows:

$\Omega: T_1 \times T_2 \times T_3 \times \ldots \times T_n = \ddot{A} \rightarrow IVIFHS^U$.

It is also defined as

$(\Omega, \ddot{A}) = \left\{\left(\check{d}, \Omega_{\ddot{A}}(\check{d})\right) : \check{d} \in \ddot{A},\ \Omega_{\ddot{A}}(\check{d}) \in IVPFS^U \in [0,1]\right\}$, where $\Omega_{\ddot{A}}(\check{d}) = \left\{\langle \zeta, \kappa_{\Omega(\check{d})}(\zeta), \delta_{\Omega(\check{d})}(\zeta)\rangle : \zeta \in U\right\}$, and $\kappa_{\Omega(\check{d})}(\zeta) = \left[\kappa^l_{\Omega(\check{d})}(\zeta), \kappa^u_{\Omega(\check{d})}(\zeta)\right]$, $\delta_{\Omega(\check{d})}(\zeta) = \left[\delta^l_{\Omega(\check{d})}(\zeta), \delta^u_{\Omega(\check{d})}(\zeta)\right]$, where $\kappa_{\Omega(\check{d})}(\zeta)$ and $\delta_{\Omega(\check{d})}(\zeta)$ represents the MD and NMD intervals, respectively, such as $\kappa^l_{\Omega(\check{d})}(\zeta), \kappa^u_{\Omega(\check{d})}(\zeta), \delta^l_{\Omega(\check{d})}(\zeta), \delta^u_{\Omega(\check{d})}(\zeta) \in [0,1]$, and $0 \leq \kappa^u_{\Omega(\check{d})}(\zeta) + \delta^u_{\Omega(\check{d})}(\zeta) \leq 1$.

The IVIFHSN can be stated as $\mathcal{F} = \left(\left[\kappa^l_{\Omega(\check{d})}(\zeta), \kappa^u_{\Omega(\check{d})}(\zeta)\right], \left[\delta^l_{\Omega(\check{d})}(\zeta), \delta^u_{\Omega(\check{d})}(\zeta)\right]\right)$.

The scoring function and accuracy function for IVIFHSS can be expressed as follows to get the alternative ranking: if $\mathcal{F} = \left(\left[\kappa^l_{\Omega(\check{d})}(\zeta), \kappa^u_{\Omega(\check{d})}(\zeta)\right], \left[\delta^l_{\Omega(\check{d})}(\zeta), \delta^u_{\Omega(\check{d})}(\zeta)\right]\right)$ be an IVIFHSN. Then

$$S(\mathcal{F}) = \frac{\kappa^l_{\Omega(\check{d})}(\zeta) + \kappa^u_{\Omega(\check{d})}(\zeta) + \delta^l_{\Omega(\check{d})}(\zeta) + \delta^u_{\Omega(\check{d})}(\zeta)}{4}$$

And

$$A(\mathcal{F}) = \frac{\left(\kappa^l_{\Omega(\check{d})}(\zeta)\right)^2 + \left(\kappa^u_{\Omega(\check{d})}(\zeta)\right)^2 + \left(\delta^l_{\Omega(\check{d})}(\zeta)\right)^2 + \left(\delta^u_{\Omega(\check{d})}(\zeta)\right)^2}{2}$$

2.5 Definition [27]

Let $\mathcal{F}_{\check{d}_k} = \left(\left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right], \left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right)$, $\mathcal{F}_{\check{d}_{11}} = \left(\left[\kappa^l_{\check{d}_{11}}, \kappa^u_{\check{d}_{11}}\right], \left[\delta^l_{\check{d}_{11}}, \delta^u_{\check{d}_{11}}\right]\right)$, and $\mathcal{F}_{\check{d}_{12}} = \left(\left[\kappa^l_{\check{d}_{12}}, \kappa^u_{\check{d}_{12}}\right], \left[\delta^l_{\check{d}_{12}}, \delta^u_{\check{d}_{12}}\right]\right)$ be three IVIFHSNs and β be a positive real number, and by algebraic norms, we have

1. $\mathcal{F}_{\check{d}_{11}} \oplus \mathcal{F}_{\check{d}_{12}} = \left(\left[\kappa^l_{\check{d}_{11}} + \kappa^l_{\check{d}_{12}} - \kappa^l_{\check{d}_{11}}\kappa^l_{\check{d}_{12}}, \kappa^u_{\check{d}_{11}} + \kappa^u_{\check{d}_{12}} - \kappa^u_{\check{d}_{11}}\kappa^u_{\check{d}_{12}}\right], \left[\delta^l_{\check{d}_{11}}\delta^l_{\check{d}_{12}}, \delta^u_{\check{d}_{11}}\delta^u_{\check{d}_{12}}\right]\right)$
2. $\mathcal{F}_{\check{d}_{11}} \otimes \mathcal{F}_{\check{d}_{12}} = \left(\left[\kappa^l_{\check{d}_{11}}\kappa^l_{\check{d}_{12}}, \kappa^u_{\check{d}_{11}}\kappa^u_{\check{d}_{12}}\right], \left[\delta^l_{\check{d}_{11}} + \delta^l_{\check{d}_{12}} - \delta^l_{\check{d}_{11}}\delta^l_{\check{d}_{12}}, \delta^u_{\check{d}_{11}} + \delta^u_{\check{d}_{12}} - \delta^u_{\check{d}_{11}}\delta^u_{\check{d}_{12}}\right]\right)$

3. $\beta\mathcal{F}_{\check{d}_k} = \left(\left[1-\left(1-\kappa^l_{\check{d}_k}\right)^\beta, 1-\left(1-\kappa^u_{\check{d}_k}\right)^\beta\right], \left[\delta^{l\ \beta}_{\check{d}_k}, \delta^{u\ \beta}_{\check{d}_k}\right]\right) = \left(1-\left(1-\left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right]\right)^\beta, \left[\delta^{l\ \beta}_{\check{d}_k}, \delta^{u\ \beta}_{\check{d}_k}\right]\right)$
4. $\mathcal{F}_{\check{d}_k}{}^\beta = \left(\left[\kappa^{l\ \beta}_{\check{d}_k}, \kappa^{u\ \beta}_{\check{d}_k}\right], \left[1-\left(1-\delta^l_{\check{d}_k}\right)^\beta, 1-\left(1-\delta^u_{\check{d}_k}\right)^\beta\right]\right) = \left(\left[\kappa^{l\ \beta}_{\check{d}_k}, \kappa^{u\ \beta}_{\check{d}_k}\right], 1-\left(1-\left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right)^\beta\right)$.

The collection of IVIFHSNs $\mathcal{F}_{\check{d}_{ij}}$, where ω_i and ν_j represent the weights assigned to professions and attributes, respectively, is considered. Let us consider a set of variables ω_i be a weight of experts where i ranges from 1 to n, such as $\omega_i > 0$, $\sum_{i=1}^n \omega_i = 1$. Additionally, let ν_j be a weight vector for sub-attributes, such as $\nu_j > 0$, $\sum_{j=1}^m \nu_j = 1$. The authors Zulqarnain et al. [28] introduced a weighted geometric aggregation operator for IVIFHSNs in the following way:

$$\text{IVIFHSWG}(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}}) = \left(\prod_{j=1}^m\left(\prod_{i=1}^n\left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}, 1-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)^{\omega_i}\right)^{\nu_j}\right)$$

The IVIFHSWG operator has been observed to yield unfavorable results under specific circumstances. In order to address such situations, the interaction AO is proposed for IVIFHSNs.

3. Weighted Geometric Interaction Aggregation Operator for IVIFHSS

We will expand IVIFHSS with specific notions and introduce the algebraic operations of IVIFHSNs in the subsequent section. Furthermore, we extend the IVIFHSIWG using algebraic operational laws.

3.1 Definition

Let $\mathcal{F}_{\check{d}_k} = \left(\left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right], \left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right)$, $\mathcal{F}_{\check{d}_{11}} = \left(\left[\kappa^l_{\check{d}_{11}}, \kappa^u_{\check{d}_{11}}\right], \left[\delta^l_{\check{d}_{11}}, \delta^u_{\check{d}_{11}}\right]\right)$, and $\mathcal{F}_{\check{d}_{12}} = \left(\left[\kappa^l_{\check{d}_{12}}, \kappa^u_{\check{d}_{12}}\right], \left[\delta^l_{\check{d}_{12}}, \delta^u_{\check{d}_{12}}\right]\right)$ be three IVIFHSNs and $\beta > 0$, the interactional algebraic norms are defined as:

1. $$\mathcal{F}_{\check{d}_{11}} \oplus \mathcal{F}_{\check{d}_{12}} = \begin{pmatrix} \left[1-\left(1-\kappa^l_{\check{d}_{11}}\right)\left(1-\kappa^l_{\check{d}_{12}}\right), 1-\left(1-\kappa^u_{\check{d}_{11}}\right)\left(1-\kappa^u_{\check{d}_{12}}\right)\right], \\ \begin{bmatrix} \left(1-\kappa^l_{\check{d}_{11}}\right)\left(1-\kappa^l_{\check{d}_{12}}\right) - \left(1-\kappa^l_{\check{d}_{11}}-\delta^l_{\check{d}_{11}}\right)\left(1-\kappa^l_{\check{d}_{12}}-\delta^l_{\check{d}_{12}}\right) \\ \left(1-\kappa^u_{\check{d}_{11}}\right)\left(1-\kappa^u_{\check{d}_{12}}\right) - \left(1-\kappa^u_{\check{d}_{11}}-\delta^u_{\check{d}_{11}}\right)\left(1-\kappa^u_{\check{d}_{12}}-\delta^u_{\check{d}_{12}}\right) \end{bmatrix} \end{pmatrix}$$
2. $$\mathcal{F}_{\check{d}_{11}} \otimes \mathcal{F}_{\check{d}_{12}} = \begin{pmatrix} \begin{bmatrix} \left(1-\delta^l_{\check{d}_{11}}\right)\left(1-\delta^l_{\check{d}_{12}}\right) - \left(1-\kappa^l_{\check{d}_{11}}-\delta^l_{\check{d}_{11}}\right)\left(1-\kappa^l_{\check{d}_{12}}-\delta^l_{\check{d}_{12}}\right), \\ \left(1-\delta^u_{\check{d}_{11}}\right)\left(1-\delta^u_{\check{d}_{12}}\right) - \left(1-\kappa^u_{\check{d}_{11}}-\delta^u_{\check{d}_{11}}\right)\left(1-\kappa^u_{\check{d}_{12}}-\delta^u_{\check{d}_{12}}\right) \end{bmatrix}, \\ \left[1-\left(1-\delta^l_{\check{d}_{11}}\right)\left(1-\delta^l_{\check{d}_{12}}\right), 1-\left(1-\delta^u_{\check{d}_{11}}\right)\left(1-\delta^u_{\check{d}_{12}}\right)\right] \end{pmatrix}$$
3. $$\beta\mathcal{F}_{\check{d}_k} = \begin{pmatrix} \left[1-\left(1-\kappa^l_{\check{d}_k}\right)^\beta, 1-\left(1-\kappa^u_{\check{d}_k}\right)^\beta\right], \\ \left[\left(1-\kappa^l_{\check{d}_k}\right)^\beta - \left(1-\kappa^l_{\check{d}_k}-\delta^l_{\check{d}_k}\right)^\beta, \left(1-\kappa^u_{\check{d}_k}\right)^\beta - \left(1-\kappa^u_{\check{d}_k}-\delta^u_{\check{d}_k}\right)^\beta\right] \end{pmatrix}$$
4. $$\mathcal{F}_{\check{d}_k}{}^\beta = \begin{pmatrix} \left[\left(1-\delta^l_{\check{d}_k}\right)^\beta - \left(1-\kappa^l_{\check{d}_k}-\delta^l_{\check{d}_k}\right)^\beta, \left(1-\delta^u_{\check{d}_k}\right)^\beta - \left(1-\kappa^u_{\check{d}_k}-\delta^u_{\check{d}_k}\right)^\beta\right], \\ \left[1-\left(1-\delta^l_{\check{d}_k}\right)^\beta, 1-\left(1-\delta^u_{\check{d}_k}\right)^\beta\right] \end{pmatrix}$$

3.2 Definition

Let $\mathcal{F}_{\check{d}_k} = \left(\left[\kappa^l_{\check{d}_k}, \kappa^u_{\check{d}_k}\right], \left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right)$ be a collection of IVIFHSNs, and ω_i and ν_j are the weight vector for experts and multi sub-parameters, respectively, with given conditions $\omega_i > 0$, $\sum_{i=1}^{n} \omega_i = 1$; $\nu_j > 0$, $\sum_{j=1}^{m} \nu_j = 1$. Then, the IVIFHSIWG operator is defined as IVIFHSIWG: $\Psi^n \longrightarrow \Psi$.

$$\text{IVIFHSIWG}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \dots\dots\dots, \mathcal{F}_{\check{d}_{nm}}\right) = \otimes_{j=1}^{m} \left(\otimes_{i=1}^{n} \left(\mathcal{F}_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}$$

3.3 Theorem

Let $\mathcal{F}_{\check{d}_{ij}} = \left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right], \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)$ be a collection of IVIFHSNs, where ($i = 1,2,3 \dots\dots, n$ *and* $j = 1,2,3, \dots\dots\dots m$) and the aggregated value is also an IVIFHSN, such as

$$\text{IVIFHSIWG}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \dots\dots\dots, \mathcal{F}_{\check{d}_{nm}}\right) = \left(\begin{array}{c} \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}, \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\right], \\ \left[\begin{array}{c} 1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \kappa^l_{\check{d}_{ij}} - \delta^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}, \\ 1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \kappa^u_{\check{d}_{ij}} - \delta^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j} \end{array}\right] \end{array}\right)$$

ω_i and ν_j show the expert's and multi sub-attributes weights, respectively, such as $\omega_i > 0, \sum_{i=1}^{n} \omega_i = 1$, $\nu_j > 0$, $\sum_{j=1}^{m} \nu_j = 1$.

Proof:

The proof of the overhead theorem can be demonstrated by mathematical induction.

For $n = 1$, we get $\omega_1 = 1$. Then,

$$\text{IVIFHSIWG}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \dots\dots\dots, \mathcal{F}_{\check{d}_{1m}}\right) = \otimes_{j=1}^{m} \left(\mathcal{F}_{\check{d}_{ij}}\right)^{\nu_j}$$

$$\text{IVIFHSIWG}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \dots\dots\dots, \mathcal{F}_{\check{d}_{1m}}\right) = \left(\left[\prod_{j=1}^{m}\left(\kappa^l_{\check{d}_{1j}}\right)^{\nu_j}, \prod_{j=1}^{m}\left(\kappa^u_{\check{d}_{1j}}\right)^{\nu_j}\right], \left[\begin{array}{c} 1 - \prod_{j=1}^{m}\left(\kappa^l_{\check{d}_{1j}}\right)^{\nu_j} - \prod_{j=1}^{m}\left(1 - \kappa^l_{\check{d}_{1j}} - \delta^l_{\check{d}_{1j}}\right)^{\nu_j}, \\ 1 - \prod_{j=1}^{m}\left(\kappa^u_{\check{d}_{1j}}\right)^{\nu_j} - \prod_{j=1}^{m}\left(1 - \kappa^u_{\check{d}_{1j}} - \delta^u_{\check{d}_{1j}}\right)^{\nu_j} \end{array}\right]\right)$$

$$= \left(\begin{array}{c} \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}, \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\right], \\ \left[\begin{array}{c} 1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1 - \kappa^l_{\check{d}_{ij}} - \delta^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}, \\ 1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1 - \kappa^u_{\check{d}_{ij}} - \delta^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j} \end{array}\right] \end{array}\right)$$

For $m = 1$, we get $\nu_1 = 1$. Then,

$$\text{IVIFHSIWG}\left(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \dots\dots\dots, \mathcal{F}_{\check{d}_{n1}}\right) = \otimes_{i=1}^{n} \left(\mathcal{F}_{\check{d}_{ij}}\right)^{\omega_i}$$

$$\mathrm{IVIFHSIWG}\left(\mathcal{F}_{\check{d}_{11}},\mathcal{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathcal{F}_{\check{d}_{n1}}\right)=\left(\begin{array}{c}\left[\prod_{i=1}^{n}\left(\kappa_{\check{d}_{i1}}^{l}\right)^{\omega_i},\prod_{i=1}^{n}\left(\kappa_{\check{d}_{i1}}^{u}\right)^{\omega_i}\right],\\ \left[\begin{array}{c}1-\left(\prod_{i=1}^{n}\left(\kappa_{\check{d}_{i1}}^{l}\right)^{\omega_i}\right)-\prod_{i=1}^{n}\left(1-\kappa_{\check{d}_{i1}}^{l}-\delta_{\check{d}_{i1}}^{l}\right)^{\omega_i},\\ 1-\left(\prod_{i=1}^{n}\left(\kappa_{\check{d}_{i1}}^{u}\right)^{\omega_i}\right)-\prod_{i=1}^{n}\left(1-\kappa_{\check{d}_{i1}}^{u}-\delta_{\check{d}_{i1}}^{u}\right)^{\omega_i}\end{array}\right]\end{array}\right)$$

$$=\left(\begin{array}{c}\left[\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(\kappa_{\check{d}_{ij}}^{l}\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(\kappa_{\check{d}_{ij}}^{u}\right)^{\omega_i}\right)^{\nu_j}\right],\\ \left[\begin{array}{c}1-\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(\kappa_{\check{d}_{ij}}^{l}\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1-\kappa_{\check{d}_{ij}}^{l}-\delta_{\check{d}_{ij}}^{l}\right)^{\omega_i}\right)^{\nu_j},\\ 1-\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(\kappa_{\check{d}_{ij}}^{u}\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^{1}\left(\prod_{i=1}^{n}\left(1-\kappa_{\check{d}_{ij}}^{u}-\delta_{\check{d}_{ij}}^{u}\right)^{\omega_i}\right)^{\nu_j}\end{array}\right]\end{array}\right)$$

So, for $n=1$ and $m=1$ the IVIFHSIWG operator is holds.

Suppose it is held for $m=\alpha_1+1, n=\alpha_2$ and $m=\alpha_1, n=\alpha_2+1$, such as

$$\otimes_{j=1}^{\alpha_1+1}\left(\otimes_{i=1}^{\alpha_2}\left(\mathcal{F}_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}=\left(\begin{array}{c}\left[\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(\kappa_{\check{d}_{ij}}^{l}\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(\kappa_{\check{d}_{ij}}^{u}\right)^{\omega_i}\right)^{\nu_j}\right],\\ \left[\begin{array}{c}1-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(\kappa_{\check{d}_{ij}}^{l}\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(1-\kappa_{\check{d}_{ij}}^{l}-\delta_{\check{d}_{ij}}^{l}\right)^{\omega_i}\right)^{\nu_j},\\ 1-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(\kappa_{\check{d}_{ij}}^{u}\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(1-\kappa_{\check{d}_{ij}}^{u}-\delta_{\check{d}_{ij}}^{u}\right)^{\omega_i}\right)^{\nu_j}\end{array}\right]\end{array}\right)$$

for $m=\alpha_1+1, n=\alpha_2+1$, we have

$$\otimes_{j=1}^{\alpha_1+1}\left(\otimes_{i=1}^{\alpha_2+1}\left(\mathcal{F}_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}=\otimes_{j=1}^{\alpha_1+1}\left(\otimes_{i=1}^{\alpha_2}\left(\mathcal{F}_{\check{d}_{ij}}\right)^{\omega_i}\otimes\left(\mathcal{F}_{\check{d}_{(\alpha_2+1)j}}\right)^{\omega(\alpha_2+1)}\right)^{\nu_j}$$

$$=\otimes_{j=1}^{\alpha_1+1}\left(\otimes_{i=1}^{\alpha_2}\left(\mathcal{F}_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\otimes_{j=1}^{\alpha_1+1}\left(\left(\mathcal{F}_{\check{d}_{(\alpha_2+1)j}}\right)^{\omega(\alpha_2+1)}\right)^{\nu_j}$$

$$\left(\begin{array}{c}\left[\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\right]\otimes\\ \left[\prod_{j=1}^{\alpha_1+1}\left(\left(\kappa^l_{\check{d}_{(\alpha_2+1)j}}\right)^{\omega_{\alpha_2+1}}\right)^{\nu_j},\prod_{j=1}^{\alpha_1+1}\left(\left(\kappa^u_{\check{d}_{(\alpha_2+1)j}}\right)^{\omega_{\alpha_2+1}}\right)^{\nu_j}\right],\\ \left[\begin{array}{c}1-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(1-\kappa^l_{\check{d}_{ij}}-\delta^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j},\\ 1-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2}\left(1-\kappa^u_{\check{d}_{ij}}-\delta^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\end{array}\right]\otimes\\ \left[\begin{array}{c}1-\prod_{j=1}^{\alpha_1+1}\left(\left(\kappa^l_{\check{d}_{(\alpha_2+1)j}}\right)^{\omega(\alpha_2+1)}\right)^{\nu_j}-\prod_{j=1}^{\alpha_1+1}\left(\left(1-\kappa^l_{\check{d}_{(\alpha_2+1)j}}-\delta^l_{\check{d}_{(\alpha_2+1)j}}\right)^{\omega(\alpha_2+1)}\right)^{\nu_j},\\ 1-\prod_{j=1}^{\alpha_1+1}\left(\left(\kappa^u_{\check{d}_{(\alpha_2+1)j}}\right)^{\omega(\alpha_2+1)}\right)^{\nu_j}-\prod_{j=1}^{\alpha_1+1}\left(\left(1-\kappa^u_{\check{d}_{ij}}-\delta^u_{\check{d}_{(\alpha_2+1)j}}\right)^{\omega(\alpha_2+1)}\right)^{\nu_j}\end{array}\right]\end{array}\right)$$

$$=\left(\begin{array}{c}\left[\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2+1}\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2+1}\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\right],\\ \left[\begin{array}{c}1-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2+1}\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2+1}\left(1-\kappa^l_{\check{d}_{ij}}-\delta^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j},\\ 1-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2+1}\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^{\alpha_1+1}\left(\prod_{i=1}^{\alpha_2+1}\left(1-\kappa^u_{\check{d}_{ij}}-\delta^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\end{array}\right]\end{array}\right)$$

Hence, it is verified that for $m=\alpha_1+1$ and $n=\alpha_2+1$. holds. Thus, the IVIFHSIWG operator holds $\forall\ m,n$.

3.4 Properties of IVIFHSIWG

If $\mathcal{F}_{\check{d}_{ij}}=\left(\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right],\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)$; $(i=1,2,3,\dots,m;j=1,2,3,\dots,n)$ be a collection of IVIFHSNs. The weights of the experts and parameters are ω_i and ν_j respectively, such as $\omega_i,\nu_j>0$, and $\sum_{i=1}^{n}\omega_i=1;\sum_{j=1}^{m}\nu_j=1$.

3.4.1 Idempotency

If $\mathcal{F}_{\check{d}_{ij}}=\mathcal{F}_{\check{d}_k}=\left(\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right],\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)\ \forall\ i,j$. Then

$$\text{IVIFHSIWG}(\mathcal{F}_{\check{d}_{11}},\mathcal{F}_{\check{d}_{12}},\dots\dots\dots,\mathcal{F}_{\check{d}_{nm}})=\mathcal{F}_{\check{d}_k}$$

Proof

As we know

$$\mathcal{F}_{\check{d}_{ij}}=\mathcal{F}_{\check{d}_k}=\left(\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right],\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)$$

Then,

$$\text{IVIFHSIWG}\left(\mathcal{F}_{\check{d}_{11}},\mathcal{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathcal{F}_{\check{d}_{nm}}\right)$$
$$=\left(\begin{array}{c}\left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa_{\check{d}_{ij}}^{l}\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa_{\check{d}_{ij}}^{u}\right)^{\omega_i}\right)^{\nu_j}\right],\\ \left[\begin{array}{c}1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa_{\check{d}_{ij}}^{l}\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\kappa_{\check{d}_{ij}}^{l}-\delta_{\check{d}_{ij}}^{l}\right)^{\omega_i}\right)^{\nu_j},\\ 1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa_{\check{d}_{ij}}^{u}\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\kappa_{\check{d}_{ij}}^{u}-\delta_{\check{d}_{ij}}^{u}\right)^{\omega_i}\right)^{\nu_j}\end{array}\right]\end{array}\right)$$
$$=\left(\begin{array}{c}\left[\left(\left(\kappa_{\check{d}_{ij}}^{l}\right)^{\sum_{i=1}^{n}\omega_i}\right)^{\sum_{j=1}^{m}\nu_j},\left(\left(\kappa_{\check{d}_{ij}}^{u}\right)^{\sum_{i=1}^{n}\omega_i}\right)^{\sum_{j=1}^{m}\nu_j}\right],\\ \left[\begin{array}{c}1-\left(\left(\kappa_{\check{d}_{ij}}^{l}\right)^{\sum_{i=1}^{n}\omega_i}\right)^{\sum_{j=1}^{m}\nu_j}-\left(\left(1-\kappa_{\check{d}_{ij}}^{l}-\delta_{\check{d}_{ij}}^{l}\right)^{\sum_{i=1}^{n}\omega_i}\right)^{\sum_{j=1}^{m}\nu_j},\\ 1-\left(\left(\kappa_{\check{d}_{ij}}^{u}\right)^{\sum_{i=1}^{n}\omega_i}\right)^{\sum_{j=1}^{m}\nu_j}-\left(\left(1-\kappa_{\check{d}_{ij}}^{u}-\delta_{\check{d}_{ij}}^{u}\right)^{\sum_{i=1}^{n}\omega_i}\right)^{\sum_{j=1}^{m}\nu_j}\end{array}\right]\end{array}\right)$$

As $\sum_{j=1}^{m}\nu_j=1$ and $\sum_{i=1}^{n}\omega_i=1$. Then,
$$=\left(\left[\kappa_{\check{d}_{ij}}^{l},\kappa_{\check{d}_{ij}}^{u}\right],\left[\begin{array}{c}1-\left(\kappa_{\check{d}_{ij}}^{l}\right)-\left(1-\kappa_{\check{d}_{ij}}^{l}-\delta_{\check{d}_{ij}}^{l}\right),\\ 1-\left(\kappa_{\check{d}_{ij}}^{u}\right)-\left(1-\kappa_{\check{d}_{ij}}^{u}-\delta_{\check{d}_{ij}}^{u}\right)\end{array}\right]\right)$$
$$=\left(\left[\kappa_{\check{d}_{ij}}^{l},\kappa_{\check{d}_{ij}}^{u}\right],\left[\delta_{\check{d}_{ij}}^{l},\delta_{\check{d}_{ij}}^{u}\right]\right)=\mathcal{F}_{\check{d}_k}.$$

3.4.2 Boundedness

Let $\mathcal{F}_{\check{d}_{ij}}=\left(\left[\kappa_{\check{d}_{ij}}^{l},\kappa_{\check{d}_{ij}}^{u}\right],\left[\delta_{\check{d}_{ij}}^{l},\delta_{\check{d}_{ij}}^{u}\right]\right)$ be a collection of IVIFHSNs where $\mathcal{F}_{\check{d}_{ij}}^{-}=\left(\left[\min_j\min_i\left(\kappa_{\check{d}_{ij}}^{l}\right),\min_j\min_i\left(\kappa_{\check{d}_{ij}}^{u}\right)\right],\left[\max_j\max_i\left(\delta_{\check{d}_{ij}}^{l}\right),\max_j\max_i\left(\delta_{\check{d}_{ij}}^{u}\right)\right]\right)$ and $\mathcal{F}_{\check{d}_{ij}}^{+}=\left(\left[\max_j\max_i\left(\kappa_{\check{d}_{ij}}^{l}\right),\underset{j}{man}\max_i\left(\kappa_{\check{d}_{ij}}^{u}\right)\right],\left[\min_j\min_i\left(\delta_{\check{d}_{ij}}^{l}\right),\min_j\min_i\left(\delta_{\check{d}_{ij}}^{u}\right)\right]\right)$, then
$\mathcal{F}_{\check{d}_{ij}}^{-}\leq\text{IVIFHSIWG}(\mathcal{F}_{\check{d}_{11}},\mathcal{F}_{\check{d}_{12}},\ldots\ldots\ldots,\mathcal{F}_{\check{d}_{nm}})\leq\mathcal{F}_{\check{d}_{ij}}^{+}$

Proof

As $\mathcal{F}_{\check{d}_{ij}}=\left(\left[\kappa_{\check{d}_{ij}}^{l},\kappa_{\check{d}_{ij}}^{u}\right],\left[\delta_{\check{d}_{ij}}^{l},\delta_{\check{d}_{ij}}^{u}\right]\right)$ be an IVIFHSN, then
$$\min_j\min_i\left(\kappa_{\check{d}_{ij}}^{l}\right)\leq\kappa_{\check{d}_{ij}}^{l}\leq\max_j\max_i\left(\kappa_{\check{d}_{ij}}^{l}\right)$$
$$\Rightarrow\left(\min_j\min_i\left(\kappa_{\check{d}_{ij}}^{l}\right)\right)^{\omega_i}\leq\left(\kappa_{\check{d}_{ij}}^{l}\right)^{\omega_i}\leq\left(\max_j\max_i\left(\kappa_{\check{d}_{ij}}^{l}\right)\right)^{\omega_i}$$
$$\Leftrightarrow\left(\min_j\min_i\left(\kappa_{\check{d}_{ij}}^{l}\right)\right)^{\sum_{i=1}^{n}\omega_i}\leq\prod_{i=1}^{n}\left(\kappa_{\check{d}_{ij}}^{l}\right)^{\omega_i}\leq\left(\max_j\max_i\left(\kappa_{\check{d}_{ij}}^{l}\right)\right)^{\sum_{i=1}^{n}\omega_i}$$
$$\Leftrightarrow\left(\min_j\min_i\left(\kappa_{\check{d}_{ij}}^{l}\right)\right)^{\sum_{j=1}^{n}\nu_j}\leq\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa_{\check{d}_{ij}}^{l}\right)^{\omega_i}\right)^{\nu_j}\leq\left(\max_j\max_i\left(\kappa_{\check{d}_{ij}}^{l}\right)\right)^{\sum_{j=1}^{n}\nu_j}$$
$$\Leftrightarrow\min_j\min_i\left(\kappa_{\check{d}_{ij}}^{l}\right)\leq\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa_{\check{d}_{ij}}^{l}\right)^{\omega_i}\right)^{\nu_j}\leq\max_j\max_i\left(\kappa_{\check{d}_{ij}}^{l}\right)\qquad\text{(a)}$$

Again, as we know that

$$\min_j \min_i \left(\kappa^u_{\check{d}_{ij}}\right) \le \kappa^u_{\check{d}_{ij}} \le \max_j \max_i \left(\kappa^u_{\check{d}_{ij}}\right)$$

$$\Rightarrow \left(\min_j \min_i \left(\kappa^u_{\check{d}_{ij}}\right)\right)^{\omega_i} \le \left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i} \le \left(\max_j \max_i \left(\kappa^u_{\check{d}_{ij}}\right)\right)^{\omega_i}$$

$$\Leftrightarrow \left(\min_j \min_i \left(\kappa^u_{\check{d}_{ij}}\right)\right)^{\sum_{i=1}^n \omega_i} \le \prod_{i=1}^n \left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i} \le \left(\max_j \max_i \left(\kappa^u_{\check{d}_{ij}}\right)\right)^{\sum_{i=1}^n \omega_i}$$

$$\Leftrightarrow \left(\min_j \min_i \left(\kappa^u_{\check{d}_{ij}}\right)\right)^{\sum_{j=1}^n \nu_j} \le \prod_{j=1}^m \left(\prod_{i=1}^n \left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j} \le \left(\max_j \max_i \left(\kappa^u_{\check{d}_{ij}}\right)\right)^{\sum_{j=1}^n \nu_j}$$

$$\Leftrightarrow \min_j \min_i \left(\kappa^u_{\check{d}_{ij}}\right) \le \prod_{j=1}^m \left(\prod_{i=1}^n \left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j} \le \max_j \max_i \left(\kappa^u_{\check{d}_{ij}}\right) \quad \text{(b)}$$

Similarly,

$$\min_j \min_i \left(\delta^l_{\check{d}_{ij}}\right) \le 1 - \prod_{j=1}^m \left(\prod_{i=1}^n \left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \kappa^l_{\check{d}_{ij}} - \delta^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j} \le \max_j \max_i \left(\delta^l_{\check{d}_{ij}}\right) \quad \text{(c)}$$

And

$$\min_j \min_i \left(\delta^u_{\check{d}_{ij}}\right) \le 1 - \prod_{j=1}^m \left(\prod_{i=1}^n \left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j} - \prod_{j=1}^m \left(\prod_{i=1}^n \left(1 - \kappa^u_{\check{d}_{ij}} - \delta^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j} \le \max_j \max_i \left(\delta^u_{\check{d}_{ij}}\right) \quad \text{(d)}$$

Let $\text{IVIFHSIWA}(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \dots \dots \dots, \mathcal{F}_{\check{d}_{nm}}) = \left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right], \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right) = \mathcal{F}_{\check{d}_{ij}}$. Then, (a), (b) and (c), (d) can be conveyed as:

$$\left[\min_j \min_i \left(\kappa^l_{\check{d}_{ij}}\right), \min_j \min_i \left(\kappa^u_{\check{d}_{ij}}\right)\right] \le \mathcal{F}_{\check{d}_k} \le \left[\max_j \max_i \left(\delta^l_{\check{d}_{ij}}\right), \max_j \max_i \left(\delta^u_{\check{d}_{ij}}\right)\right]$$ and

$$\left[\max_j \max_i \left(\kappa^l_{\check{d}_{ij}}\right), \operatorname{man}_j \max_i \left(\kappa^u_{\check{d}_{ij}}\right)\right] \le \mathcal{F}_{\check{d}_k} \le \left[\min_j \min_i \left(\delta^l_{\check{d}_{ij}}\right), \min_j \min_i \left(\delta^u_{\check{d}_{ij}}\right)\right]$$ respectively.

By score function

$$S(\mathcal{F}_{\check{d}_k}) = \frac{\kappa^l_{\check{d}_k} + \kappa^u_{\check{d}_k} + \delta^l_{\check{d}_k} + \delta^u_{\check{d}_k}}{4}$$
$$\le \left[\max_j \max_i \left(\kappa^l_{\check{d}_{ij}}\right), \operatorname{man}_j \max_i \left(\kappa^u_{\check{d}_{ij}}\right)\right] - \left[\min_j \min_i \left(\delta^l_{\check{d}_{ij}}\right), \min_j \min_i \left(\delta^u_{\check{d}_{ij}}\right)\right]$$
$$= S\left(\mathcal{F}^-_{\check{d}_k}\right)$$

$$S(\mathcal{F}_{\check{d}_k}) = \frac{\kappa^l_{\check{d}_k} + \kappa^u_{\check{d}_k} + \delta^l_{\check{d}_k} + \delta^u_{\check{d}_k}}{4}$$
$$\ge \left[\min_j \min_i \left(\kappa^l_{\check{d}_{ij}}\right), \min_j \min_i \left(\kappa^u_{\check{d}_{ij}}\right)\right] - \left[\max_j \max_i \left(\delta^l_{\check{d}_{ij}}\right), \max_j \max_i \left(\delta^u_{\check{d}_{ij}}\right)\right]$$
$$= S\left(\mathcal{F}^+_{\check{d}_k}\right)$$

We get the following states using the order relation among IVIFHSNs.

$\mathcal{F}^-_{\check{d}_k} \le \text{IVIFHSIWG}(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \dots \dots \dots, \mathcal{F}_{\check{d}_{nm}}) \le \mathcal{F}^+_{\check{d}_k}$.

3.4.3 Shift invariance

Let $\mathcal{F}_{\check{d}_k}=\left(\left[\kappa^l_{\check{d}_k},\kappa^u_{\check{d}_k}\right],\left[\delta^l_{\check{d}_k},\delta^u_{\check{d}_k}\right]\right)$ be an IVIFHSN. Then
$IVIFHSIWG\ (\mathcal{F}_{\check{d}_{11}}\otimes\mathcal{F}_{\check{d}_k},\mathcal{F}_{\check{d}_{12}}\otimes\mathcal{F}_{\check{d}_k},\dots\dots\dots,\mathcal{F}_{\check{d}_{nm}}\otimes\mathcal{F}_{\check{d}_k})=$
$IVIFHSIWG\ (\mathcal{F}_{\check{d}_{11}},\mathcal{F}_{\check{d}_{12}},\dots\dots\dots,\mathcal{F}_{\check{d}_{nm}})\otimes\mathcal{F}_{\check{d}_k}$.

Proof

As $\mathcal{F}_{\check{d}_k}=\left(\left[\kappa^l_{\check{d}_k},\kappa^u_{\check{d}_k}\right],\left[\delta^l_{\check{d}_k},\delta^u_{\check{d}_k}\right]\right)$ and $\mathcal{F}_{\check{d}_{ij}}=\left(\left[\kappa^l_{\check{d}_{ij}},\kappa^u_{\check{d}_{ij}}\right],\left[\delta^l_{\check{d}_{ij}},\delta^u_{\check{d}_{ij}}\right]\right)$ be two IVIFHSNs. Then, using definition 3.1 (2)

$$\mathcal{F}_{\check{d}_k}\otimes\mathcal{F}_{\check{d}_{ij}}=\begin{pmatrix}\begin{bmatrix}\left(1-\delta^l_{\check{d}_k}\right)\left(1-\delta^l_{\check{d}_{ij}}\right)-\left(1-\kappa^l_{\check{d}_k}-\delta^l_{\check{d}_{ij}}\right)\left(1-\kappa^l_{\check{d}_k}-\delta^l_{\check{d}_{ij}}\right),\\ \left(1-\delta^u_{\check{d}_k}\right)\left(1-\delta^u_{\check{d}_{ij}}\right)-\left(1-\kappa^u_{\check{d}_k}-\delta^u_{\check{d}_{ij}}\right)\left(1-\kappa^u_{\check{d}_{ij}}-\delta^u_{\check{d}_{ij}}\right)\end{bmatrix},\\ \left[1-\left(1-\delta^l_{\check{d}_k}\right)\left(1-\delta^l_{\check{d}_{ij}}\right),1-\left(1-\delta^u_{\check{d}_k}\right)\left(1-\delta^u_{\check{d}_{ij}}\right)\right]\end{pmatrix}$$

So,
$IVIFHSIWG\ (\mathcal{F}_{\check{d}_{11}}\otimes\mathcal{F}_{\check{d}_k},\mathcal{F}_{\check{d}_{12}}\otimes\mathcal{F}_{\check{d}_k},\dots\dots\dots,\mathcal{F}_{\check{d}_{nm}}\otimes\mathcal{F}_{\check{d}_k})$
$=\otimes^m_{j=1}\left(\otimes^n_{i=1}\left(\mathcal{F}_{\check{d}_{ij}}\otimes\mathcal{F}_{\check{d}_k}\right)^{\omega_i}\right)^{\nu_j}$

$$=\begin{pmatrix}\left[\prod_{j=1}^m\left(\prod_{i=1}^n\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\left(\kappa^l_{\check{d}_k}\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^m\left(\prod_{i=1}^n\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\left(\kappa^u_{\check{d}_k}\right)^{\omega_i}\right)^{\nu_j}\right],\\ \begin{bmatrix}1-\prod_{j=1}^m\left(\prod_{i=1}^n\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\left(\kappa^l_{\check{d}_k}\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\kappa^l_{\check{d}_{ij}}-\delta^l_{\check{d}_{ij}}\right)^{\omega_i}\left(1-\kappa^l_{\check{d}_k}-\delta^l_{\check{d}_k}\right)^{\omega_i}\right)^{\nu_j},\\ 1-\prod_{j=1}^m\left(\prod_{i=1}^n\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\left(\kappa^u_{\check{d}_k}\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\kappa^u_{\check{d}_{ij}}-\delta^u_{\check{d}_{ij}}\right)^{\omega_i}\left(1-\kappa^u_{\check{d}_k}-\delta^u_{\check{d}_k}\right)^{\omega_i}\right)^{\nu_j}\end{bmatrix}\end{pmatrix}$$

$$=\begin{pmatrix}\left[\kappa^l_{\check{d}_k}\prod_{j=1}^m\left(\prod_{i=1}^n\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j},\kappa^u_{\check{d}_k}\prod_{j=1}^m\left(\prod_{i=1}^n\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\right],\\ \begin{bmatrix}1-\left(\kappa^l_{\check{d}_k}\right)\prod_{j=1}^m\left(\prod_{i=1}^n\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}-\left(1-\kappa^l_{\check{d}_k}-\delta^l_{\check{d}_k}\right)\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\kappa^l_{\check{d}_{ij}}-\delta^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j},\\ 1-\left(\kappa^u_{\check{d}_k}\right)\prod_{j=1}^m\left(\prod_{i=1}^n\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}-\left(1-\kappa^u_{\check{d}_k}-\delta^u_{\check{d}_k}\right)\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\kappa^u_{\check{d}_{ij}}-\delta^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\end{bmatrix}\end{pmatrix}$$

$$=\begin{pmatrix}\left[\prod_{j=1}^m\left(\prod_{i=1}^n\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j},\prod_{j=1}^m\left(\prod_{i=1}^n\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\right],\\ \begin{bmatrix}\prod_{j=1}^m\left(\prod_{i=1}^n\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\kappa^l_{\check{d}_{ij}}-\delta^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j},\\ \prod_{j=1}^m\left(\prod_{i=1}^n\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}-\prod_{j=1}^m\left(\prod_{i=1}^n\left(1-\kappa^u_{\check{d}_{ij}}-\delta^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\end{bmatrix}\end{pmatrix}\otimes\left(\left[\kappa^l_{\check{d}_k},\kappa^u_{\check{d}_k}\right],\left[\delta^l_{\check{d}_k},\delta^u_{\check{d}_k}\right]\right)$$

$=IVIFHSIWG\ (\mathcal{F}_{\check{d}_{11}},\mathcal{F}_{\check{d}_{12}},\dots\dots\dots,\mathcal{F}_{\check{d}_{nm}})\otimes\mathcal{F}_{\check{d}_k}$.

3.4.4 Homogeneity

Prove that IVIFHSIWG $(\beta\mathcal{F}_{\check{d}_{11}}, \beta\mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \beta\mathcal{F}_{\check{d}_{nm}}) = \beta$ IVIFHSIWG $(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}})$ for any $\beta > 0$.

Proof

Let $\mathcal{F}_{\check{d}_{ij}} = \left(\left[\kappa^l_{\check{d}_{ij}}, \kappa^u_{\check{d}_{ij}}\right], \left[\delta^l_{\check{d}_{ij}}, \delta^u_{\check{d}_{ij}}\right]\right)$ be an IVIFHSN and $\beta > 0$. Then, using definition 3.1, we have

$$\mathcal{F}_{\check{d}_k}{}^{\beta} = \left(\left[{\kappa^l_{\check{d}_k}}^{\beta}, {\kappa^u_{\check{d}_k}}^{\beta}\right], \left[1-\left(1-\delta^l_{\check{d}_k}\right)^{\beta}, 1-\left(1-\delta^u_{\check{d}_k}\right)^{\beta}\right]\right)$$

$$= \left(\left[{\kappa^l_{\check{d}_k}}^{\beta}, {\kappa^u_{\check{d}_k}}^{\beta}\right], 1-\left(1-\left[\delta^l_{\check{d}_k}, \delta^u_{\check{d}_k}\right]\right)^{\beta}\right)$$

So,

IVIFHSIWG $(\beta\mathcal{F}_{\check{d}_{11}}, \beta\mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \beta\mathcal{F}_{\check{d}_{nm}})$

$$= \left(\begin{array}{c} \left[\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa^l_{\check{d}_{ij}}\right)^{\beta\omega_i}\right)^{\nu_j}, \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa^u_{\check{d}_{ij}}\right)^{\beta\omega_i}\right)^{\nu_j}\right], \\ \left[\begin{array}{c} 1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa^l_{\check{d}_{ij}}\right)^{\beta\omega_i}\right)^{\nu_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\kappa^l_{\check{d}_{ij}}-\delta^l_{\check{d}_{ij}}\right)^{\beta\omega_i}\right)^{\nu_j}, \\ 1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa^u_{\check{d}_{ij}}\right)^{\beta\omega_i}\right)^{\nu_j} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\kappa^u_{\check{d}_{ij}}-\delta^u_{\check{d}_{ij}}\right)^{\beta\omega_i}\right)^{\nu_j} \end{array}\right] \end{array}\right)$$

$$= \left(\begin{array}{c} \left[\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\right)^{\beta}, \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\right)^{\beta}\right], \\ \left[\begin{array}{c} 1-\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\right)^{\beta} - \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\kappa^l_{\check{d}_{ij}}-\delta^l_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\right)^{\beta}, \\ 1-\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\kappa^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\right)^{\beta} - \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\kappa^u_{\check{d}_{ij}}-\delta^u_{\check{d}_{ij}}\right)^{\omega_i}\right)^{\nu_j}\right)^{\beta} \end{array}\right] \end{array}\right)$$

$= \beta$ IVIFHSIWG $(\mathcal{F}_{\check{d}_{11}}, \mathcal{F}_{\check{d}_{12}}, \ldots\ldots\ldots, \mathcal{F}_{\check{d}_{nm}})$.

4. Multi-Criteria Group Decision-Making Approach Using IVIFHSIWG Operator

To confirm the effects of intentional geometric interaction AO, a DM method is settled to confiscate MCGDM limitations. Also, numerical descriptions for material selection are presented to demonstrate the projected technique's suitability.

4.1 Proposed MCGDM Approach

Consider $\mathfrak{I} = \{\mathfrak{I}^1, \mathfrak{I}^2, \mathfrak{I}^3, \ldots, \mathfrak{I}^s\}$ and $\mathcal{U} = \{\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \ldots, \mathcal{U}_r\}$ be the set of substitutes and specialists separately. The weights of specialists are specified as $\omega_i = (\omega_1, \omega_2, \omega_3, \ldots, \omega_n)^T$ such that $\omega_i > 0$, $\sum_{i=1}^{n}\omega_i = 1$. Let $\mathfrak{L} = \{e_1, e_2, e_3, \ldots, e_m\}$ be the set of attributes with their compatible multi sub-attributes such as $\mathfrak{L}' = \{(e_{1\rho} \times e_{2\rho} \times \ldots \times e_{m\rho})$ for all $\rho \in \{1, 2, \ldots, t\}\}$ with weights $\nu = (\nu_1, \nu_2, \nu_3, \ldots, \nu_n)^T$ such that $\nu_i > 0$, $\sum_{i=1}^{n}\nu_i = 1$. and can be indicated as $\mathfrak{L}' = \{\check{d}_{\partial}: \partial \in \{1, 2, \ldots, m\}\}$. The group of specialists $\{\kappa^i$: i = 1, 2,…, $n\}$ consider the alternatives $\{\mathfrak{H}^{(z)}$: z = 1, 2, …, $s\}$ under the preferred sub-attributes $\{\check{d}_{\partial}$: ∂ = 1, 2, …, $k\}$ in the form of IVIFHSNs, such as $\left(\mathfrak{I}^{(z)}_{\check{d}_{ik}}\right)_{n\times m}$ =

$\left(\left[\kappa^{l}_{\check{d}_{ik}}, \kappa^{u}_{\check{d}_{ik}}\right], \left[\delta^{l}_{\check{d}_{ik}}, \delta^{u}_{\check{d}_{ik}}\right]\right)_{n*m}$. Where $0 \leq \kappa^{l}_{\check{d}_{ik}}, \kappa^{u}_{\check{d}_{ik}}, \delta^{l}_{\check{d}_{ik}}, \delta^{u}_{\check{d}_{ik}} \leq 1$ and $0 \leq \left(\kappa^{u}_{\check{d}_{ik}}\right)^2 + \left(\delta^{u}_{\check{d}_{ik}}\right)^2 \leq 1$ for all i, k. The rules of the operator-based algorithm established are as follows:
Step 1: Specialist's advice on each alternative in the form of IVIFHSNs.

$$\left(\mathfrak{I}^{(z)}_{\check{d}_{ik}}\right)_{n\times m} = \left(\left[\kappa^{l}_{\check{d}_{ik}}, \kappa^{u}_{\check{d}_{ik}}\right], \left[\delta^{l}_{\check{d}_{ik}}, \delta^{u}_{\check{d}_{ik}}\right]\right)_{n\times m}$$

$$= \begin{bmatrix} \left(\left[\kappa^{l}_{\check{d}_{11}}, \kappa^{u}_{\check{d}_{11}}\right], \left[\delta^{l}_{\check{d}_{11}}, \delta^{u}_{\check{d}_{11}}\right]\right) & \left(\left[\kappa^{l}_{\check{d}_{12}}, \kappa^{u}_{\check{d}_{12}}\right], \left[\delta^{l}_{\check{d}_{12}}, \delta^{u}_{\check{d}_{12}}\right]\right) & \cdots & \left(\left[\kappa^{l}_{\check{d}_{1m}}, \kappa^{u}_{\check{d}_{1m}}\right], \left[\delta^{l}_{\check{d}_{1m}}, \delta^{u}_{\check{d}_{1m}}\right]\right) \\ \left(\left[\kappa^{l}_{\check{d}_{21}}, \kappa^{u}_{\check{d}_{21}}\right], \left[\delta^{l}_{\check{d}_{21}}, \delta^{u}_{\check{d}_{21}}\right]\right) & \left(\left[\kappa^{l}_{\check{d}_{22}}, \kappa^{u}_{\check{d}_{22}}\right], \left[\delta^{l}_{\check{d}_{22}}, \delta^{u}_{\check{d}_{22}}\right]\right) & & \left(\left[\kappa^{l}_{\check{d}_{2n}}, \kappa^{u}_{\check{d}_{2n}}\right], \left[\delta^{l}_{\check{d}_{2m}}, \delta^{u}_{\check{d}_{2m}}\right]\right) \\ \vdots & & \vdots & \vdots \\ \left(\left[\kappa^{l}_{\check{d}_{n1}}, \kappa^{u}_{\check{d}_{n1}}\right], \left[\delta^{l}_{\check{d}_{n1}}, \delta^{u}_{\check{d}_{n1}}\right]\right) & \left(\left[\kappa^{l}_{\check{d}_{n2}}, \kappa^{u}_{\check{d}_{n2}}\right], \left[\delta^{l}_{\check{d}_{n2}}, \delta^{u}_{\check{d}_{n2}}\right]\right) & \cdots & \left(\left[\kappa^{l}_{\check{d}_{nm}}, \kappa^{u}_{\check{d}_{nm}}\right], \left[\delta^{l}_{\check{d}_{nm}}, \delta^{u}_{\check{d}_{nm}}\right]\right) \end{bmatrix}$$

Step 2: Generate a normalization decision matrix using the normalization rule for each alternative by altering the cost-type parameters to the benefit type.

$$\mathcal{F}_{\check{d}_{ik}} = \begin{cases} \mathcal{F}^{c}_{\check{d}_{ij}} = \left(\left[\delta^{l}_{\check{d}_{ik}}, \delta^{u}_{\check{d}_{ik}}\right], \left[\kappa^{l}_{\check{d}_{ik}}, \kappa^{u}_{\check{d}_{ik}}\right]\right)_{n\times m} & cost\ type\ parameter \\ \mathcal{F}_{\check{d}_{ij}} = \left(\left[\kappa^{l}_{\check{d}_{ik}}, \kappa^{u}_{\check{d}_{ik}}\right], \left[\delta^{l}_{\check{d}_{ik}}, \delta^{u}_{\check{d}_{ik}}\right]\right)_{n\times m} & benefit\ type\ parameter \end{cases}$$

Step 3: Aggregated values are calculated by the IVIFHSIWG operator for each alternative.
Step 4: Use the score function to calculate the score value for each alternative.
Step 5: Govern the most appropriate alternative.
Step 6: Alternatives rank.

The flow chart of the above-presented model is prearranged in Figure 1.

4.3. Application of IVIFHSIWG Operator in Material Selection

According to the Consulate General Committee on Environment Change, risky environmental mortification arises due to social achievements. Climate change has important natural implications, including animal extinction [29], low agricultural production [30], extra-sensational weather conditions [31] and social movement [32]. There is a growing impetus to reduce overall greenhouse gas emissions to reduce climate change corridors. Recently, France sanctioned a prerequisite for 40% of greenhouse gas emissions parallel to 1990 by 2030 [33]. Until carbon gas schedules are not individual producers of greenhouse gases. The environmental security organization circulated a description for 76 % of all social emissions from the United States [34]. A massive reduction in greenhouse gas radiation earnings fewer use of vestige energies. However, since its discovery, it has not been an easy job. The energy formed from hydrocarbons is transporters and important energy sources. It will be incorporated into a generally approachable system to make a considerable impression of decarbonization. In 2017, fossil fuels accounted for more than 85% of worldwide energy formation [35].

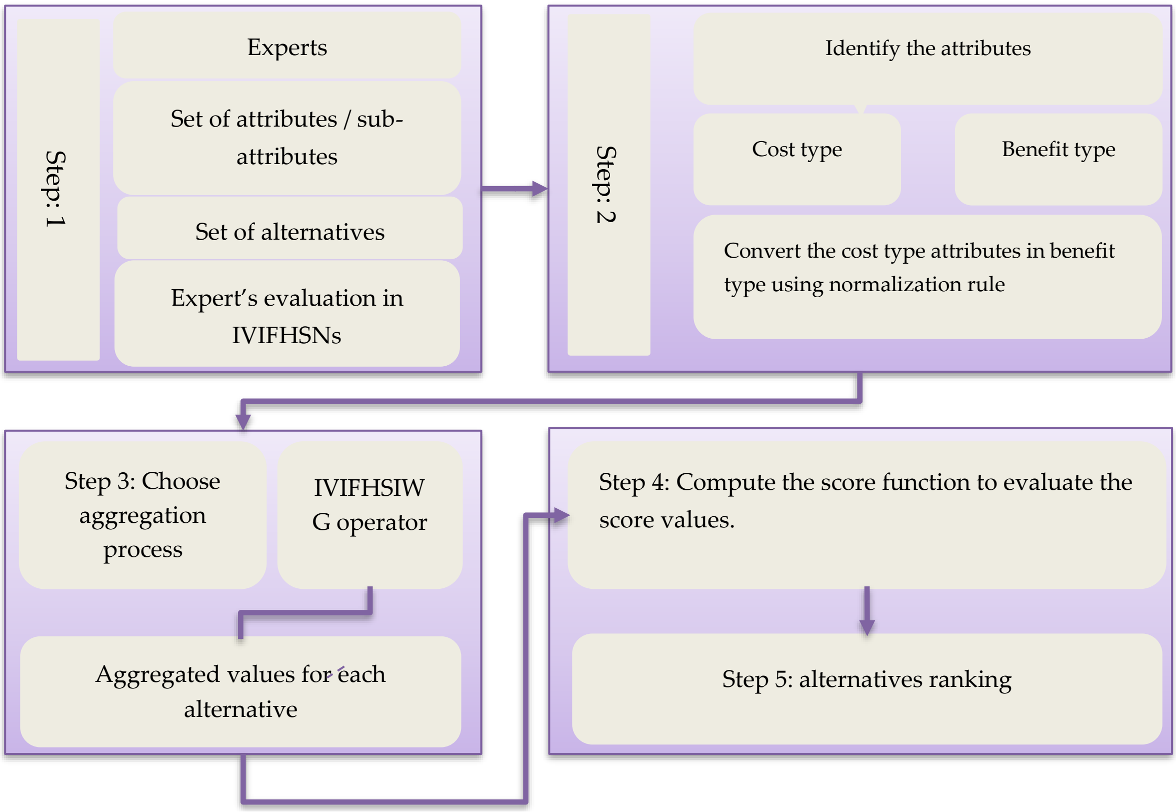

Fig. 1. Flow chart of the proposed MCGDM technique

So, if the world had to completely change to eliminate the hydrogen budget for the invitation of fossil fuels, the energy shortage would be solved immediately. This factor provides an important function when determining the proper power supply. However, this study does not defend this matter. As impermanence approaches the end of the era of 'low-cost oil,' where there are particular concessions in discipline and power manufacturing, the discovery of different energy exporters is crucial. Strict mitigation mechanisms will be the ultimate best option for countries dealing with hydrogen. Hydrogen, which can be conceived as a harmonizing energy source in automobiles, impacts engineering revolutions, such as hydrogen fuel cells, to deliver direct fuel for the fabrication of automatic transmission and interior ignition engines CO2. One of the characteristics of its driving force is the trend of hydrogen production in changes in the feedstock of its construction. Since there is almost no lavish hydrogen in the environment. The only option is to proclaim it from the organic connections of other scraps. Two behaviors yield hydrogen and other properties: angry hydrocarbons or cracked water. The process of condensed fermentation is employed for the purpose of eliminating low-molecular-weight hydrocarbons. The behavior of water can be influenced by various factors such as environmental conditions, temperature, and energy utilization. One proposed method for generating hydrogen from water in a novel manner involves the combustion of coal during a water scarcity event.

This approach effectively updates the utilization of fossil-based surplus sources of energy, specifically natural gas, which used to be converted into hydrogen. In general, while fossil fuels are widely utilized in industrial sectors and face restrictions owing to their contribution to warming the planet, alternative forms of energy are expected to emerge as the most prominent alternatives, mostly driven by economic and environmental factors. In strength representation, the just-designed hydrogen fuel differs from the commonly used hydrogen fuel in satisfactory weight and capacity. This hydrogen is inconsiderately connected using its energy potential as the most important residual feature. The amount of energy per kilogram of hydrogen is 120 MJ. Hydrogen has a small volume energy density accompanied by its admirable gravity compactness. The state of its deposition determines the thickness of hydrogen. The fluid density of up to 700 bar is not enough, like a considerable number of hydrocarbons of gasoline and diesel. Only liquescent hydrogen can disturb a representative quantity, static less than a quarter of the expanse of fuel. In this manner, hydrogen storage vessels designed for portable applications will supersede the conventional use of fluid hydrocarbon ampoules [36]. Cryogenic storage tanks are sometimes referred to as cryogenic preserving containers. The wall bottle is a thermally insulated container with a double-walled construction designed to provide excellent heat resistance. This fluid carries oxygen, nitrogen, hydrogen, helium, and argon at temperatures < 110 K/163 °C. Since water is just an extra gas, it is incredibly non-toxic and environmentally safe when restored to electricity. The components used by cryogenic vessels depend on safety and budget [37]. In essence, cryogenic containers play a safety concern and corporate circumstance. From this point of view, minor temperature disturbances can be described as follows:

Fracture toughness: The fracture hardness of molten nitrogen is around -196 °C, while the evaporation point of liquid nitrogen is in close proximity. The temperature of hydrogen is approximately -253 °C. It is not flexible enough and is difficult to change. Therefore, sufficient supplies are strong enough to withstand unbendable flaws. Face-centered cubic iron meshes are appropriate because they are not affected by minor heat. All nickel contains an additional 7% nickel for constructing copper compounds, aluminum, their compounds, and austenitic stainless steel storage cryogenic containers [38]. Heat transfer: The transient heat at the obstruction of the cryogenic container is generally conductive. Choose a component with low, hot air conductivity.

Thermal stress: The internal cordon shrinks due to the little heat, triggering warm air stress. Therefore, a component with minor thermal conductivity is appropriate. Thermal diffusion: In routine, communal thermal remoteness is preposterous. The material should be designated with such a strategy that it can dissipate heat quickly.

Material classification is a critical corporate stage in any manufacturing sector. Manufacturing companies are designed by targeting, budgeting, and environmental protection targets and are often made of insufficient materials. The conventional merchandise plan is to select the most appropriate content design standards only if exciting demonstrations are on the lowest possible budget. Material selection is made by looking at many conflicting DM processes. In this case, the current AOs are instigated by different theories. These AOs should be restructured to discourse about these observable concerns. We plan to take some innovative phases and escort interaction geometric AO to assemble numerous IVIFHSNs. The ideal product we predicted surpassed other models. All configurations can be classified according to the above specifications and DM concepts. The case study was presented at a motor portion manufacturing firm in Malaysia and showed cryogenic storage of electrical components. As part of applied sustainability, corporations must indicate the suitable materials for the details they produce. It first focuses on cryogenic storage vessels, DM parameters, and second-factor weight inputs to collect the material. The IVIFHSS model and the proposed interaction geometric AO are used to astounding convolution and hesitation in social finding. The MS evokes the fundamental support of sustainable development: materials should be proper, environmentally pleasing, and beneficial to humanity.

The selection process begins with an initial showing of the material used for the console, drawing in the application's alignment structure. During the selection process, a potentially suitable fabric is determined. It takes seriously the content that decides which initial MS can be used for the dashboard development. Four ingredients are designated, and then capacity is checked: $\mathfrak{I}^1$= Ti–6Al–4V, $\mathfrak{I}^2$= SS301–FH, $\mathfrak{I}^3$ = 70Cu–30Zn, and $\mathfrak{I}^4$ = Inconel 718.

The feature of the material collection is indicated as follows: $\mathfrak{L}$ = {$d_1 =$ Specific gravity = managing information around the consideration of tenacities of several materials, $d_2 =$ Toughness index, $d_3 =$ Yield stress, $d_4 =$ Easily accessible}. The conforming sub-attributes of the deliberated factors, Specific gravity = managing information around the consideration of tenacities of several materialsd_1 ={$d_{11} =$ assess corporal variations, $d_{12} =$ govern the degree of regularity among tasters} Toughness index = d_2 = {d_{21} = Charpy V − Notch Impact Energy, d_{22} = Plane Strain Fracture Toughness}, Yield stress = d_3 = {d_{31} = Yield stress}, Easily accessible = d_4 = {d_{41} = Easily accessible}. Let $\mathfrak{L}'$ = $d_1 \times d_2 \times d_3 \times d_4$ be a set of sub-attributes

$\mathfrak{L}' = d_1 \times d_2 \times d_3 \times d_4 = \{d_{11}, d_{12}\} \times \{d_{21}, d_{22}\} \times \{d_{31}\} \times \{d_{41}\}$

$= \begin{Bmatrix} (d_{11}, d_{21}, d_{31}\ d_{41}), (d_{11}, d_{22}, d_{31}, d_{41}), \\ (d_{12}, d_{21}, d_{31}, d_{41}), (d_{12}, d_{22}, d_{31}, d_{41}) \end{Bmatrix}$, $\mathfrak{L}' = \{\check{d}_1, \check{d}_2, \check{d}_3, \check{d}_4\}$ represents the multi-sub-attributes with weights $(0.3, 0.1, 0.2, 0.4)^T$. Let $\{\mathcal{U}_1, \mathcal{U}_2, \mathcal{U}_3, \mathcal{U}_4\}$ be a team of four specialists with weights $(0.1, 0.2, 0.4, 0.3)^T$. To decide the best alternative, experts present their priorities in IVIFHSNs form, which are given in Tables 1-4.

Step 1: The expert's advice on each alternative, in the form of IVIFHSNs, is given in Table 1- 4.

Table 1

Expert's opinion in the form of IVIFHSN for $\mathfrak{J}^1$

	$\check{d}_1$	$\check{d}_2$	$\check{d}_3$	$\check{d}_4$
$\mathcal{U}_1$	([0.4, 0.5], [0.2, 0.5])	([0.2, 0.4], [0.5, 0.6])	([0.1, 0.3], [0.2, 0.5])	([0.2, 0.4], [0.2, 0.6])
$\mathcal{U}_2$	([0.2, 0.4], [0.2, 0.6])	([0.1, 0.3], [0.4, 0.5])	([0.2, 0.3], [0.3, 0.7])	([0.2, 0.4], [0.2, 0.5])
$\mathcal{U}_3$	([0.3, 0.5], [0.1, 0.4])	([0.4, 0.5], [0.2, 0.4])	([0.4, 0.5], [0.3, 0.4])	([0.2, 0.6], [0.2, 0.4])
$\mathcal{U}_4$	([0.4, 0.6], [0.3, 0.4])	([0.1, 0.3], [0.3, 0.6])	([0.3, 0.4], [0.3, 0.5])	([0.3, 0.4], [0.3, 0.5])

Table 2

Expert's opinion in the form of IVIFHSN for $\mathfrak{J}^2$

	$\check{d}_1$	$\check{d}_2$	$\check{d}_3$	$\check{d}_4$
$\mathcal{U}_1$	([0.3, 0.4], [0.5, 0.5])	([0.2, 0.4], [0.4, 0.5])	([0.2, 0.4], [0.4, 0.5])	([0.4, 0.5], [0.3, 0.5])
$\mathcal{U}_2$	([0.3, 0.5], [0.3, 0.4])	([0.1, 0.4], [0.4, 0.5])	([0.1, 0.5], [0.3, 0.4])	([0.4, 0.5], [0.3, 0.4])
$\mathcal{U}_3$	([0.2, 0.6], [0.1, 0.4])	([0.1, 0.2], [0.2, 0.8])	([0.4, 0.5], [0.3, 0.5])	([0.3, 0.6], [0.2, 0.4])
$\mathcal{U}_4$	([0.2, 0.3], [0.3, 0.6])	([0.3, 0.5], [0.1, 0.4])	([0.3, 0.4], [0.2, 0.6])	([0.1, 0.3], [0.3, 0.6])

Table 3

Expert's opinion in the form of IVIFHSN for $\mathfrak{J}^3$

	$\check{d}_1$	$\check{d}_2$	$\check{d}_3$	$\check{d}_4$
$\mathcal{U}_1$	([0.3, 0.4], [0.2, 0.5])	([0.3, 0.4], [0.4, 0.6])	([0.3, 0.4], [0.4, 0.5])	([0.3, 0.4], [0.3, 0.6])
$\mathcal{U}_2$	([0.4, 0.6], [0.3, 0.4])	([0.2, 0.5], [0.2, 0.3])	([0.3, 0.5], [0.3, 0.5])	([0.2, 0.6], [0.2, 0.4])
$\mathcal{U}_3$	([0.2, 0.4], [0.3, 0.5])	([0.3, 0.4], [0.3, 0.6])	([0.3, 0.5], [0.3, 0.4])	([0.1, 0.3], [0.4, 0.5])
$\mathcal{U}_4$	([0.3, 0.6], [0.3, 0.4])	([0.3, 0.5], [0.2, 0.4])	([0.2, 0.5], [0.3, 0.4])	([0.3, 0.4], [0.3, 0.6])

Table 4

Expert's opinion in the form of IVIFHSN for $\mathfrak{J}^4$

	$\check{d}_1$	$\check{d}_2$	$\check{d}_3$	$\check{d}_4$
$\mathcal{U}_1$	([0.3, 0.5], [0.2, 0.4])	([0.2, 0.6], [0.1, 0.4])	([0.2, 0.5], [0.3, 0.4])	([0.3, 0.4], [0.4, 0.5])
$\mathcal{U}_2$	([0.2, 0.7], [0.1, 0.3])	([0.1, 0.5], [0.4, 0.5])	([0.3, 0.5], [0.4, 0.5])	([0.2, 0.5], [0.3, 0.4])
$\mathcal{U}_3$	([0.2, 0.5], [0.1, 0.4])	([0.2, 0.5], [0.1, 0.5])	([0.2, 0.4], [0.2, 0.6])	([0.3, 0.5], [0.1, 0.5])
$\mathcal{U}_4$	([0.2, 0.4], [0.5, 0.5])	([0.2, 0.5], [0.2, 0.4])	([0.2, 0.4], [0.3, 0.6])	([0.2, 0.5], [0.4, 0.5])

Step 2: There is no need for normalization as all parameters exhibit similarity.

Step 3: The IVIFHSIWG operator generates the accumulated information for every option.

$\Theta_1 = ([0.4724, 0.5648], [0.1537, 0.5004])$, $\Theta_2 = ([0.4626, 0.6951], [0.4906, 0.6952])$, $\Theta_3 = ([0.5326, 0.7471], [0.4960, 0.6764])$, and $\Theta_4 = ([0.5169, 0.6123], [0.5219, 0.6145])$.

Step 4: Use the score function to calculate the score value for each alternative, such as $S(\Theta_1) = 0.4228$, $S(\Theta_2) = 0.5859$, $S(\Theta_3) = 0.6130$, and $S(\Theta_4) = 0.5664$.

Step 5: In the above calculation, we get the classification of substitutes $S(\Theta_3) > S(\Theta_2) > S(\Theta_4) > S(\Theta_1)$. Govern the most appropriate alternative $\mathfrak{J}^3$ is the most acceptable alternative.

Step-6: So, $\mathfrak{J}^3 > \mathfrak{J}^2 > \mathfrak{J}^4 > \mathfrak{J}^1$ is the gained rank of substitutes.

5. Comparative Studies

To authorize the realism of the anticipated methodology, the following section plans to compare the proposed model with the prevalent methods.

5.1 Supremacy of the Planned Approach

The planned methodology enables representative verdicts to be made in the DM system. We presented the MCGDM method by the IVIFHSIWG operator. Our presented MCGDM approach delivers the most delicate and detailed facts about DM obstacles. The planned model is versatile and talkative, consistent with changing volatility, obligation, and efficiency. Disparate models have explicit ordering procedures; therefore, there are straight variations in the organization of the desired procedure to their prospects. The study and assessment of this system resolute that the consequences gained by traditional approaches were falsely equivalent to hybrid structures. Also, due to auspicious circumstances, several merged configurations of FS, such as IVFS, IVIFS, and IVIFSS, are concerted in IVIFHSS. It is comfortable to combine inadequate and equivocal facts in the DM process. The figures of this case can be designated more precisely and realistically. So, our proposed technique is extra effectual, more evocative, enhanced, and superior to numerous hybrid FS organizations. Table 5 below analyzes the proposed approach and the characteristics of some prevailing theories.

Table 5
Characteristic analysis of different models with a projected model

	membership	Non-membership	attributes	attributes in intervals form	sub-attributes of any attribute	Interactional aggregation
IVFS [2]	✓	×	×	✓	×	×
IVIFS [7]	✓	✓	×	✓	×	×
IFSS [18]	✓	✓	✓	×	×	×
IVIFSS [20]	✓	✓	✓	✓	×	×
IFHSS [26]	✓	✓	✓	×	✓	×
IVIFHSS [28]	✓	✓	✓	✓	✓	×
Proposed IVIFHSIWG	✓	✓	✓	✓	✓	✓

5.2 Comparative Analysis

To demonstrate the usefulness of the scheduled technique, we associate the results obtained with some predominant methods under IVIFS, IVIFSS, and IVIFHSS. Table 6 summarizes all the values. Xu and Gou [9] point out that the IVIFWG operator, Wei [39] IVIFOWG operator, and Wang and Liu [40] IVIFEWG operator cannot compute parametric standards for alternatives. Moreover, if experts deliberate that the sum of MD and NMD is more than 1, then the above AO cannot adapt to this situation. Zulqarnain et al. [21] introduced the AOs for IVIFSS to handle the mentioned above scenario. However, these AOs are incapable of contracting multi sub-attributes of deliberated parameters.

Meanwhile, Zulqarnain et al. [28] resolved complications by extending the AOs for IVIFHSS. Still, these AOs operators cannot handle the uncertain scenario whenever the experts consider the interaction. According to the investigation, the current geometric AOs deliver undesirable results in some cases. In order to navigate the intricacies associated with the subject matter, we have successfully established the geometric interaction AO for IVIFHSS. This approach is capable of accommodating several sub-attributes that are equivalent to the prevalent geometric AOs. Based on the information above, the operator discussed in this article is deemed highly significant, consistent, and rich. Table 6 presents an evaluation of the projected model in comparison to the existing models.

Table 6
Comparison of intentional operators with some prevalent operators

AO	$\mathfrak{J}^1$	$\mathfrak{J}^2$	$\mathfrak{J}^3$	$\mathfrak{J}^4$	Alternatives ranking	Optimal choice
IVIFWG[9]	0.3681	0.2116	0.4509	0.3573	$\mathfrak{J}^3 > \mathfrak{J}^1 > \mathfrak{J}^4 > \mathfrak{J}^2$	$\mathfrak{J}^3$
IVIFOWG [39]	0.3104	0.2753	0.3914	0.2952	$\mathfrak{J}^3 > \mathfrak{J}^1 > \mathfrak{J}^4 > \mathfrak{J}^2$	$\mathfrak{J}^3$
IVIFEWG [40]	0.0235	0.0253	0.0723	0.0584	$\mathfrak{J}^3 > \mathfrak{J}^4 > \mathfrak{J}^2 > \mathfrak{J}^1$	$\mathfrak{J}^3$
IVIFSWG [21]	0.2365	0.3734	0.7134	0.5428	$\mathfrak{J}^3 > \mathfrak{J}^4 > \mathfrak{J}^2 > \mathfrak{J}^1$	$\mathfrak{J}^3$
IVIFHSWG [28]	0.3929	0.4319	0.5188	0.5094	$\mathfrak{J}^3 > \mathfrak{J}^4 > \mathfrak{J}^2 > \mathfrak{J}^1$	$\mathfrak{J}^3$
IVIFHSIWG	0.4228	0.5859	0.6130	0.5664	$\mathfrak{J}^3 > \mathfrak{J}^2 > \mathfrak{J}^4 > \mathfrak{J}^1$	$\mathfrak{J}^3$

5.3 Advantages of Proposed Research

In the subsequent section, we will explain the assistance of the planning methodology.

i. The planned approach combines the idea of parametrization with IVIFHSS to address the importance of DM constraints. The degree of intuitionistic consistency parametrization somewhat the possibility of having greeting and justification. Along with these aspects, this correspondence has the fantastic ability to compute computationally and interpolate the universe in terms of effective presentations.
ii. The model places significant emphasis on the thorough evaluation of parameter outcomes and their corresponding sub-parameters, hence assisting decision-makers in making well-balanced and coherent conclusions regarding DM.

The thorough validation of all forms and aspects of the prominent theory renders any inclination to regard it as a generic assemblage of prevailing concepts unwarranted.

6. Conclusions

Decision-making is a systematic procedure that involves the deliberate organization and selection of prioritized options from a range of alternatives. Decision-making (DM) is a complex and multifaceted procedure characterized by its ability to transition between several perspectives or alternatives. The importance of employing an instinctive method in isolating quantitative decision-makers is well recognized. The most effective approach in decision-making is to demonstrate close dedication and focused attention toward one's aims. In real DM, alternative facts told by assessment professionals are consistently inaccurate, unbalanced, and stimulating. So, IVIFHSNs can be used to contest this unreliable information. This work's core objective is to prolong geometric invariably.

AO's interaction for IVIFHSS. First, we discuss the algebraic interactional laws for IVIFHSNs. In keeping with the established algebraic interactional operational rules, we present the IVIFHSIWG operator and its basic features. Furthermore, a DM approach is intended to handle the complexity of

MCGDM built on a renowned operator. To demonstrate the compensations of the settled technique, we present an inclusive mathematical illustration of MS in the industrial sector. Finally, based on the consequences extended, it was indomitable that the methodology planned in this study is the most concrete and effective for resolving MCGDM obstructions related to the prevalent approaches.

Author Contributions

Conceptualization, S.A. and H.N.; methodology, S.A.; software, H.N.; validation, I.S. and R.M.Z.; formal analysis, R.M.Z; investigation, I.S.; resources, H.N.; data curation, H.N.; writing—original draft preparation, S.A.; writing—review and editing, R.M.Z.; visualization, I.S.; supervision, R.M.Z.; project administration, R.M.Z. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability Statement

The datasets generated during and/or analysed during the current study are not publicly available due to the privacy-preserving nature of the data. However, they can be obtained from the corresponding author upon reasonable request.

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